Jadavpur University

ME (Power Engg.) First Yr. 1st Semester 204-2025

ADVANCED MATHEMATICAL AND COMPUTATIONAL TECHNIQUES

Total Time: 3 hrs. **Answer each part in a separate answer script** **Full Marks: 100**

PART I

(70 Marks: Answer All Questions)

1. Compute the Extrema for the function $f(x,y) = ye^x - 3x - y + 2$. CO(1) 10

   Deduce the locus of all he local Maxima and Minima of the function
   $f(x,y) = (x^2 + y^2)e^{-(x^2+y^2)}$. CO(2) 20

   OR

   Develop the MATLAB code to generate the plot for a hollow sphere with centre at (0,0,0) and radius of 3 units using concept of *constrained optimization.* CO(2) 20

2. Using the concept of Legrange Multipliers establish the maximum and minimum values of $f(x,y) = (x-\alpha)^2 + (y-\beta)^2$ given the constraint $g(x,y) = x^2 + y^2 = c^2$ where α, β are positive real numbers. CO(2) 20

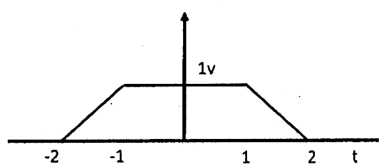
   OR

   Considering
   $\frac{d^2u(x,t)}{dt^2} = c^2 \frac{d^2u(x,t)}{dx^2}$ $\forall\, 0 < x < l, t > 0,$ $u(0,t) = u(l,t) = 0\ \forall t > 0$ and $u(x,0) = sin(5\pi x) + 2sin(7\pi x), u_t(x,0) = 0 \forall\ 0 < x < l$, using the separation of variables method , deduce $u(x,t)\ \forall t > 0$ for a string of length l fixed at both ends with $u(x,t)$ representing transverse vibrations. CO(2) 20

3. For a Gamma Function $\Gamma(x)$ prove the relationship $\Gamma(x+1) = x\Gamma(x)$. CO(4) 10
   Hence compute the value of $\Gamma(1.5)$ CO(4) 10

   OR

   Prove the relationship $J_{-n}(x) = (-1)^n J_n(x)$ CO(4) 10
   Derive the Fourier Transform for the signal shown in the Figure below CO(4) 10



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PART-II

Full Marks: 30

Group-1 (CO1)

Question No.1 is compulsory and answers any two (2) from the rest questions.

Marks: 3 x 10 =30

1.Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ **(CO1)**

Or

Solve the equations $3x+y+2z = 3$, $2x - 3y - z = - 3$, $x+2y + z = 4$ by Gauss elimination method.

2. Find the directional derivative of $f = x^2 - y^2 + 2z^2$ at the point P(1, 2, 3) in the direction of the line PQ where Q is the point (5, 0, 4). (CO3)

3.State Green's theorem.. Use this theorem to solve the given function $F = (x - xy) + y^2$ when a particle moves counter clockwise along the rectangle whose vertices are given as (0, 0), (4, 0), (4, 6) and (0, 6). (CO3)

4.Find Div. *F* and Curl *F*, where $F = \text{Grad}\,(x^3 + y^3 + z^3 - 3xyz)$ (CO3)