Ref. No. : Ex/PG/ME/T/127A/2025

M.E. MECHANICAL ENGINEERING FIRST YEAR SECOND SEMESTER EXAM – 2025

Subject: ROTOR DYNAMICS **Time: 3 Hours** **Full Marks: 100**

Answer any four questions.

Any missing information can be assumed suitably with appropriate justification.

Q1. (a) Derive the equations of motion (in polar coordinate system) for lateral vibration of a Jeffcott rotor with mass-unbalance, mounted on two identical rigid bearings. Consider that m is the mass of the rotor, e is the unbalance eccentricity, k is the shaft stiffness, c is the constant damping coefficient against the vibration along both the radial and the transverse directions and Ω is the constant spin speed of the rotor about the *X* axis which is perpendicular to the rotor-disc. The force due to weight can be assumed as negligible compared to the force due to unbalance. **Clearly state other assumptions relevant to this derivation.** Explain what do you by the terms 'the whirling radius' and 'the speed of whirling'. **[10]**

(b) From the above equations of motion, obtain the **expressions of whirling radius and its phase** with respect to mass-unbalance vector for the condition of **steady state synchronous whirling**. Also deduce the **expression for external torque** required to drive this rotor at steady state. **[8]**

(c) Show that the magnitude of steady state whirling radius will be maximum at spin speed equal to $\dfrac{\sqrt{k/m}}{\sqrt{1-2\varsigma^2}}$, where $\varsigma = \dfrac{c}{2\sqrt{km}}$ **[7]**

Q2. Consider a rotor disc is subject to gyroscopic action during vibration under the influence of unbalance excitation at a constant angular speed Ω about its axis, and consequently, in addition to the later vibration (characterized by v and w, the components of the translational motion of its geometric centre along *Y*- and *Z*-directions respectively), it undergoes small angular oscillation (characterized by B and Γ, the small angles of rotation about *Y*- and *Z*-directions respectively). Suitably **define** and **use the Euler's angles and the corresponding derivatives to derive the total angular velocity vector for this system using unit vectors of a body-fixed coordinate system (i.e., the coordinate axes fixed to the body of the disc),** which is initially aligned with inertial coordinate system *X*-*Y*-*Z*. During the derivation **clearly show different orientations of the body-fixed axes system (i.e., the axes fixed to the disc) and their transformation relationship**. **Obtain the expressions of the kinetic energy of the disc due to both translational and rotational vibration**. Assume that m be the mass of the disc, J_T be its transverse mass moment of inertia, J_P be its polar mass moment of inertia, e be the unbalance eccentricity. **[25]**

Q3. (a) With neat sketches and stating the appropriate assumptions deduce the Reynold's equations for a fluid film bearing in the form: $\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(h^3\frac{\partial p}{\partial z}\right)=6\eta U\frac{\partial h}{\partial x}+12\eta\frac{\partial h}{\partial t}$ where h be the local film thickness, p be the local fluid pressure, η be the fluid viscosity and U be the surface speed of the journal within the bearing. ***X***- and ***Z***-coordinates are corresponding to the circumferential and axial directions of the bearing respectively. **[12]**-

(b) From the above equation obtain an approximate simplified form of Reynold's equation for a long plain cylindrical journal bearing operating at steady state. **[3]**

(c) Consider a Jeffcott rotor mounted on a pair of identical journal bearings at its ends. Assume standard symbols for bearing stiffness and damping coefficients, spin speed etc. Find an expression of energy dissipation per cycle of rotation for such a rotor during its steady state vibration due to unbalance excitation in terms of the bearing stiffness and damping coefficients, spin speed and amplitudes of steady state response components of journal centre. **[10]**

Q4. (a) Consider a Jeffcott rotor mounted on rigid bearings and running at a constant rotating speed with both non-rotating external damping and rotating internal damping of viscous nature acting on it. **Deduce the equations of motion for such a rotor for its transverse vibration in inertial Cartesian coordinates**. Consider that m be the mass of the rotor, e be the unbalance eccentricity, k be the shaft stiffness, c_e and c_i be the constant external and internal damping coefficient against the vibration, Ω be the constant spin speed of the rotor. Deduce and expression of **stability limit of the spin speed** for such a rotor. **[15]**

(b) Derive an expression of **overall transfer matrix for free torsional vibration** of a torsionally compliant rotor having two discs and mounted on a pair of radial bearings as shown in **Fig. Q4b**. The torsional degree of freedom of the rotor is unrestricted at both the ends. The torsional stiffness and polar mass moment of inertia of different parts are same as shown. Clearly indicate in a figure different nodes and sections of the system. Also clearly show the expressions of various field and point matrices during the derivation. From the overall transfer matrix find also the appropriate characteristics equation for free vibration of the system. **[10]**

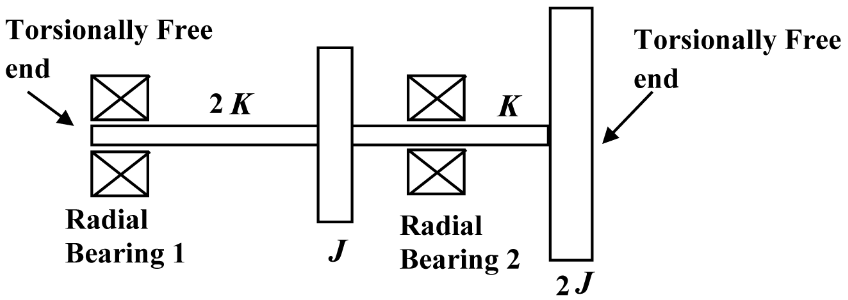

Fig. Q4b

Q5. (a) In the context of a rigid rotor write short notes, with relevant sketches, on (i) static-unbalance, (ii) quasi-static unbalance, (iii) couple unbalance, (iv) dynamic unbalance. **[10]**

(b) With relevant sketches explain the **single-plane balancing** of a short rigid rotor using influence coefficient method. **[15]**