**MASTER OF MECHANICAL ENGINEERING EXAMINATION, 2025**

($1^{st}$ Year, $1^{st}$ Semester)

**STRESS AND DEFORMATION ANALYSIS**

Time: **03 hours** Full Marks: **100**

Different parts of a question must be answered together.
Provide sketches wherever applicable.
Missing data, if any, may be appropriately assumed.
**Answer any Four (4))**

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1(*a*). Define State of Stress at a point in a loaded body. Explain what do you understand by Principal Stresses and Principal planes. Prove that the all the principal stresses (as obtained from the solution of the cubic characteristic equation) are real in nature. Write down the form of the stress invariants when the state of stress is referred to the principal axes. [02+03+06+02]

1(*b*). Prove: $\tau^2 = n_x^2 n_y^2(\sigma_1 - \sigma_2)^2 + n_y^2 n_z^2(\sigma_2 - \sigma_3)^2 + n_z^2 n_x^2(\sigma_3 - \sigma_1)^2$ [where, $\sigma_1$, $\sigma_2$ and $\sigma_3$ are the principal stresses and $n_x$, $n_y$, $n_z$ are the direction cosines] [04]

1(*c*). The state of stress at a point in a material is given as follows: [05+03]

$$\begin{bmatrix} 40 & 20 & 10 \\ 20 & -30 & 35 \\ 10 & 35 & 40 \end{bmatrix} \quad \text{(in MPa)}$$

Find out the normal and shear stresses on a plane with outward drawn normal having the direction cosines: $n_x = 0.924$, $n_y = 0.383$, $n_z = 0$. Resolve the state of stress shown into a hydrostatic stress state and a state of pure shear.

2(*a*). The principal stresses at a given point ($P$) of a stressed body are $\sigma_1$, $\sigma_2$ and $\sigma_3$; while, the frame of reference is chosen along the principal axes. Schematically draw the Mohr circle for the stress state and show the important points on the circles. Prove that when the points with coordinates ($\sigma$, $\tau$) corresponding to all possible planes passing through the point $P$ are marked in the $\sigma$-$\tau$ plane, these points are bounded by the Mohr circles. [03+02+08]

2(*b*). Considering the stress components on a small rectangular element, carry out the following: [12]
(*i*). Prove that the cross-shears are equal
(*ii*). Derive the differential equations of equilibrium (consider body forces)

3(*a*). The displacement field of a body is given as follows: $u = (x^2 + 2z)$; $v = (4x + 2y^2 + z)$ and $w = 4z^2$.
(*i*). Determine the strain components at point (2,2,3). Show the change in results if strain-displacement relations are assumed to be linear.
(*ii*). Determine the strain of the point in the direction of line joining the point from origin.
(*iii*). Also determine the strain of the point in the direction given by the direction cosines (0, $1/\sqrt{2}$, $1/\sqrt{2}$).
[For (*ii*) and (*iii*) use linear strain-displacement relations] [08+04+03]

3(*b*). Define stress concentration. Describe theoretical stress concentration factor and fatigue stress concentration factor. Develop a relation between these two types of stress concentration factors. [02+03+03]

3(*c*). Provide an explanation for the following statement - "Stress concentration factors may not be required at all for ductile materials under static loading, but stress concentration factors must be used for brittle materials subjected to static loads." [02]

4(*a*). What is meant by failure of a machine component – explain. In the context of design, describe the importance of failure theories. [02+03]

4(*b*). Write down the statement of Octahedral Shear Stress Theory and obtain an expression for failure criterion according to this theory (derive the expression for octahedral shear stress necessary to obtain the failure criterion). Considering bi-axial state of stress, show the region of safety for this failure theory. [03+05+02]

4(*c*). A shaft of 30 mm diameter is subjected to simultaneous action of an axial tensile load of 120 kN and a torque of 300 N-m. The material is plain C-steel having tensile yield strength of 250 MPa. Assuming the loads are static and steady, calculate the factor of safety for the shaft according to maximum shear stress theory and distortion energy theory. [10]

5(*a*). Explain what is meant by constitutive relations. [03]

5(*b*). For a homogeneous isotropic linearly elastic material, write down the constitutive relations that express strain in terms of stresses in matrix form. Also write down the constitutive relations that express stresses in terms of strains in matrix form. In this context, identify the stiffness and the compliance matrix. What is the relation between these two matrices? [03+03+02+01]

5(*c*). How many independent elastic constants are needed to define the constitutive relations for an orthotropic material? If the material is isotropic in nature, how many independent elastic constants are sufficient to describe the material behaviour? Justify the answer with proof. [01+01+05]

5(*d*). Express the elastic modulus and Poisson ratio in terms Lame's constants. [06]

6(*a*). Write short notes on the following topics: [$4 \times 05 = 20$]

(*i*). Sensitivity of a strain gauge

(*ii*). Null and deflection method of strain measurement

(*iii*). Hamilton's Principle

(*iv*). Assumptions of Euler beam theory

6(*b*). Derive the general differential equation of a column. [05]

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