Ref. No.: Ex/Arch/Math/T/114/2025

Bachelor of Architecture Examination 2025

(First Year, First Semester)

MATHEMATICS - I

Time : Three Hours **Full Marks : 100**

Use separate answer script for each Part.

Symbols / Notations have their usual meanings.

Part I (50 Marks)

Answer Question No. 1 and any six from the rest

1. State Rolle's theorem. Give the geometrical interpretation of this theorem. Verify Rolle's theorem for the function $f(x) = (x-a)^m(x-b)^n$; m, n are positive integers. [2+2+4]

2. Using Lagrange's mean value theorem, show that

$$\frac{v-u}{1+v^2} < tan^{-1}v - tan^{-1}u < \frac{v-u}{1+u^2},\ 0 < u < v$$ [7]

3. If $y = e^{asin^{-1}x}$ then using Leibnitz's theorem show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$$ [7]

4. Use Maclaurin's theorem to show that $sin^{-1}x = x + \frac{1}{2}\frac{x^3}{3} + \frac{1.3}{2.4}\frac{x^5}{5} + \frac{1.3.5}{2.4.6}\frac{x^7}{7} + \cdots$ and hence find the value of π. [5+2]

5. Evaluate $\lim_{x\to 0}\left(\frac{1}{x^2} - \frac{1}{sin^2x}\right)$ [7]

6. If $u = log(x^3+y^3+z^3-3xyz)$ then show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$$ [7]

7. State and prove Euler theorem on homogeneous function of x and y of degree n. [2+5]

8. If $u = tan^{-1}\frac{x^3+y^3}{x-y}$, show that
$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = sin4u - sin2u$$ [7]

9. If $f(x)$ is maximum or minimum at $x = c$, then $f'(c) = 0$ if it exists. Is the condition sufficient? Justify your answer with an example.
Determine the points of maxima or minima of the function $f(x) = x^5 - 5x^4 + 5x^3 + 12$.
[2+5]

10. Find the dimensions of the right circular cone of minimum volume which can be circumscribed about a sphere of radius 8 cm. [7]

Ref. No.: Ex/Arch/Math/T/114/2025

BACHELOR OF ARCHITECTURE ENGINEERING EXAMINATION, 2025

(1st Year, 1st Semester)

Mathematics-I

Time:Three hours | Full Marks: 100

(50 marks for each Part)

(Symbols and notations have their usual meanings)

Use a separate Answer-Script for each Part

PART-II (50 Marks)

Answer*Q. No.1* and any ***three*** from the rest

Answer the following questions:

1. Evaluate $\int_0^{\frac{\pi}{2}} sin^m x \, \mathrm{dx} \times \int_0^{\frac{\pi}{2}} sin^{m+1} x \, \mathrm{dx}$, where m>-1. **5**

2. Examine the convergence of the following integrals:

   a) $\int_2^{\infty} \frac{dx}{\log x}$ b) $\int_0^1 \frac{1}{(x+1)(x+2)\sqrt{x(1-x)}} dx$ c) $\int_1^{\infty} \frac{(x+\sqrt{1+x})}{x^2+99\sqrt[5]{1+x^4}} dx$ **15**

3. a) Evaluate $\iiint \frac{dxdydz}{\sqrt{1-x^2-y^2}}$, the field of integration being the positive octant of the sphere $x^2+y^2+z^2=1$.

   b) Evaluate $\iint_D e^{\frac{y-x}{y+x}} \, dxdy$, where D is the region bounded by the triangle with vertices (0, 0), (1, 0), (0, 1). **8+7**

4. a) Calculate the value of $\int_0^1 \sqrt{1-x^3} \, dx$ using Simpson's $\frac{1}{3}$ rule by taking eight intervals.

   b) Using limit of a sum show that, $\int_0^{\frac{\pi}{2}} \sin x \, dx = 1$.

   c) Compute the length of one arch of the cycloid $\mathrm{x} = \mathrm{a}(\theta - \sin\theta), \mathrm{y} = \mathrm{a}(1-\cos\theta)$. **5+5+5**

5. a) Find the volume of the solid obtained by revolving the cardioid $\mathrm{r} = \mathrm{a}(1+\cos\theta)$ about x-axis.

   b) Let f:[-3, 3]→ R be define by $f(x) = \begin{cases} 2x\sin\frac{\pi}{x} - \pi\cos\frac{\pi}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$.

   Examine whether f is Riemann integral in [-3, 3] and hence find $\int_{-3}^{3} f \, dx$.

   c) What do you mean by the convergence of improper integral $\int_{-\infty}^{\infty} f(x) \, dx$ **8+5+2**

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