Ref. No.: **Ex/ET/PC/B/T/323/2025**

B.E. ELECTRONICS AND TELE-COMMUNICATION ENGINEERING THIRD YEAR SECOND SEMESTER - 2025

Subject: DIGITAL CONTROL SYSTEMS Time: 3 Hours Full Marks: 100

All parts of the same question must be answered at one place only.

PART-A: Answer any ONE [CO1]

1. State and prove Nyquist sampling theorem. 10

2. (a) A sampler cannot be described by a unique transfer function. Justify. 3

(b) A signal $g(t)$ band-limited to B Hz is sampled by a periodic pulse train $p_T(t)$ made up of a rectangular pulse of width $1/8B$ seconds (centered at the origin) repeating at the Nyquist rate ($2B$ pulses per second). Show that the sampled signal is given by 7

$$\bar{g}(t)=\frac{1}{4}g(t)+\sum_{n=1}^{\infty}\left[\frac{2}{n\pi}\sin\left(\frac{n\pi}{4}\right)g(t)\cos\left(4\pi nBt\right)\right]$$

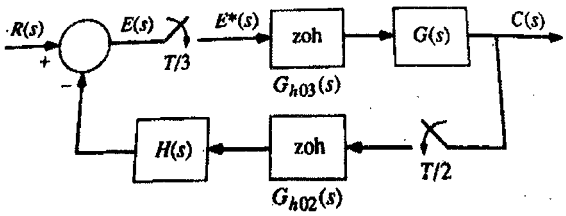
PART-B: Answer any TWO [CO2]

3. (a) Describe how the order of a hold circuit determines the extent of smoothening of a sampled signal. 4

(b) Derive the transfer function of a polygonal hold circuit. 6

4. Determine the output response of a slow-fast sampling system by realizing a fast sampler (with sampling period T/N) using a slow sampler (of sampling period T). 10

5. Determine the closed loop transfer function for the following multi-rate control system. 10



PART-C: Answer any TWO [CO3]

6. Evaluate the closed loop transfer function of the following antenna control system with $D(z) = 1$, $T = 0.05$ s, $K = 20$. 15

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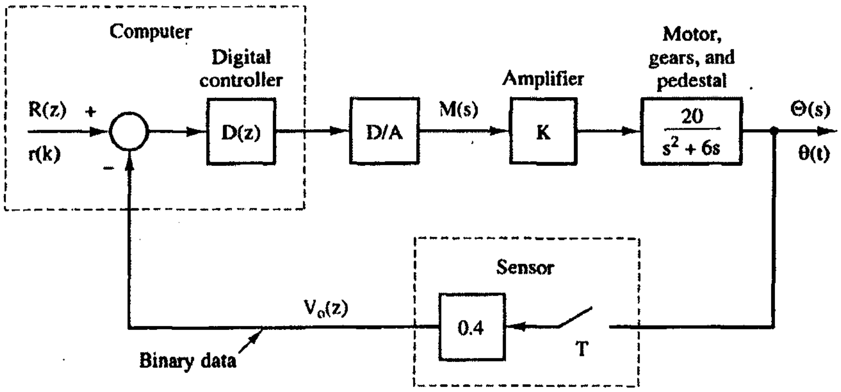
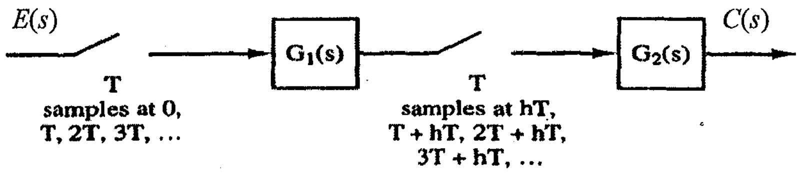
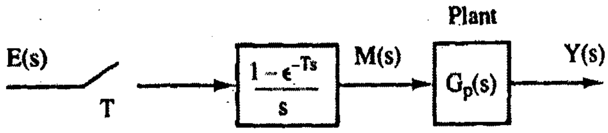


7. Determine $C(z)$ of the following system. 15



8. Consider the following system with the behavior of the plant described by the differential equation

$$\frac{d^2y(t)}{dt^2}+0.15\frac{dy(t)}{dt}+0.005y(t)=0.1m(t).$$



(a) Draw a continuous-time simulation diagram for $G_p(s)$ and give the state equations. 7

(b) Use the state-variable model of part (a) to find a discrete state model for the entire system. 8

9. (a) Derive the expression of maximum overshoot of a second-order closed loop digital control system. 10

(b) For a discrete time control system with transfer function 5

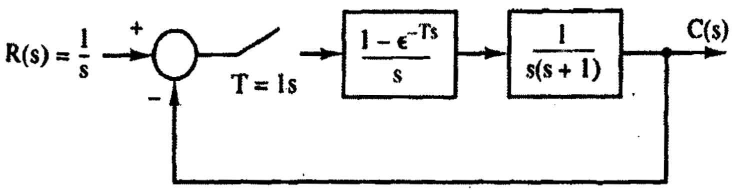
$$\frac{Y(z)}{R(z)}=\frac{0.368z+0.264}{z^2-z+0.632},\quad T=1\text{ sec}$$

determine the damping ratio for the corresponding s-plane second order transfer function.

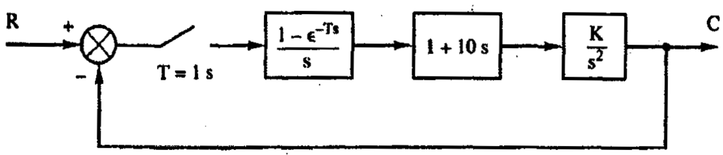
PART-D: Answer any TWO [CO4]

10. (a) What is bilinear transform? 4

(b) Determine the marginal stability condition of the following system using Jury's test. 6



11. Find the range of K for stability of the system from its root locus. Also determine the oscillating frequency for the marginal stability. 10



12. State and prove Nyquist stability criterion for digital control system. 10

13. (a) Explain the effect of addition of open loop poles on the closed loop stability using root locus. 6

(b) Using Nyquist stability criteria, comment on stability of a closed loop system with open loop transfer function $\overline{GH}(z) = \dfrac{0.632Kz}{(z-1)(z-0.368)}$. 4

PART-E: Answer any TWO [CO5]

14. For a plant described by 10

$$\vec{x}(k+1) = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} \vec{x}(k) + \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix} u(k)$$

find the gain matrix K required to realize a damping ratio of 0.46 and a time constant of 0.5 s.

15. Derive the state dynamics and hence the transfer function of a state observer. 10

16. Determine the control law $u(k)$ that minimizes 10

$$J_2 = \sum_{k=0}^{2} \left(x^2(k) + u^2(k) \right)$$

for the plant given by $x(k+1) = 2x(k) + u(k)$.

17. Consider a linear digital control system described by 10

$$\vec{x}(k+1) = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.2 \end{bmatrix} \vec{x}(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k).$$

Find the optimal control $u^o(k)$ so that the Lyapunov function $V(\vec{x}) = \vec{x}^T(k)\mathbf{P}\vec{x}(k)$ is minimized where $\mathbf{P}$ is a positive definite solution of $\mathbf{A}^T\mathbf{P}\mathbf{A} - \mathbf{P} = -\mathbf{I}$.