Ref. No.: Ex/ME(M2)/BS/B/MATH/T/111/2025

B.E. MECHANICAL ENGINEERING EXAMINATIONS - 2025

First Year First Semester

Mathematics-I

Time : Three hours Full Marks:100

(Notations and symbols have their usual meanings.

Use separate answer script for each group.)

Group-A

Answer any five questions from the followings.

1. (a) State the Sandwich theorem. Show that if $x_n = \left(\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \cdots + \frac{1}{\sqrt{n^2+n}}\right)$, then the sequence $\{x_n\}$ converges to 1.

1+3

(b) Define bounded sequence. Show that if $x_n = \frac{n^2+3n}{2n^2+n-1}$, then the sequence $\{x_n\}$ converges to $\frac{1}{2}$.

1+3

(c) Examine the following sequence for convergence:

$$x_n = 3 + (-1)^n$$

2

2. (a) Examine the convergence of the following series:

i. $\frac{5}{1.2.4} + \frac{7}{2.3.5} + \frac{9}{3.4.6} + \cdots$

ii. $\frac{1}{3} + \left(\frac{2}{5}\right)^2 + \left(\frac{3}{7}\right)^3 + \cdots$

iii. $\sum_{n=1}^{\infty} \frac{1}{n^{(1+\frac{1}{n})}}$

3+3+3

(b) What is the necessary condition of convergence of a positive term series?

[Turn over

1

3. (a) Find the values of p and q such that,

$$\lim_{x\to 0}\frac{x(1-px)+q\sin x}{x^3}=\frac{1}{3}$$

5

(b) Evaluate:

i. $\lim\limits_{x\to 0}\left(\frac{1}{x}+\frac{1}{\sin x}\right)$

ii. $\lim\limits_{x\to 0}\cos x\ \log\left(\frac{1+x}{1-x}\right)$

2+3

4. (a) State Lagrange's mean value theorem. Give the geometrical significance of LMVT.

2+1

(b) State Cauchy's mean value theorem. If f(x) = e^x and g(x) = e^{-x}, then use Cauchy's MVT to prove that there exists c which is arithmetic mean between a and b. 2+3

(c) Find the value of $(1001)^{\frac{1}{3}}$ with the help of LMVT.

2

5. (a) Find the radius of curvature of $x^3+y^3=3xy$ at $(\frac{3}{2}a,\ \frac{3}{2}a)$

3

(b) Find the asymptotes of the curve $x=\frac{t^2+1}{t^2-1}, y=\frac{t^2}{t-1}$.

7

6. (a) If $y=(\sin^{-1}x)^2$, prove that $(1-x^2)y_{n+2}-x(2n+1)y_{n+1}-n^2y_n=0$.

5

(b) Given that z is a function x and y and also $x=u+v; y=uv$, Prove that, $\frac{\partial^2 z}{\partial u^2}-2\frac{\partial^2 z}{\partial u\partial v}+\frac{\partial^2 z}{\partial v^2}=(x^2-4y)\frac{\partial^2 z}{\partial y^2}-2\frac{\partial z}{\partial y}$.

5

7. (a) Define directional derivative for a real valued function defined in $\mathbb{R}^2$ at some direction $u=<a,b>$. 3

(b) A function defined as,

$$F(x,y)=\begin{cases}\frac{xy}{(x^2+y^2)} &, (x,y)\neq(0,0)\\ 0 &, (x,y)=(0,0)\end{cases}$$

Show that the partial derivatives exist at (0,0) but the function is not continuous at (0,0).

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8. (a) Find the absolute maximum and minimum values of the function $f(x, y) = x^2 - 2xy + 2y$ on the rectangle $D = \{(x, y)|0 \le x \le 3; 0 \le y \le 2\}$.

5

(b) Find the extreme values of the function $f(x, y) = x^2 + 2y$ on the circle $x^2 + y^2 = 1$. 5

GROUP- B

Answer any five questions from the following.

1. (a) State fundamental theorem of Integral Calculus.
(b) A function is defined on [0,1] by

$$f(x) = \begin{cases} x & \text{x is rational} \\ 0, & \text{x is irrational} \end{cases}$$

Find the upper and lower integral sums corresponding to the partition P of [0,1] into n equal sub intervals and show that f is not Riemann integrable on [0,1].
(c) Show that $\frac{\sqrt{3}}{8} \le \int_{\pi/4}^{\pi/3} \frac{sinx}{x} dx \le \frac{\sqrt{2}}{6}$ $2 + 4 + 4$

2. (a) Prove that $\sqrt{\pi}\Gamma(2x) = 2^{2x-1}\Gamma(x)\Gamma(x + 1/2)$, for $x > 0$
(b) Examine the convergence of
(i) $\int_1^\infty \frac{dx}{x^{\frac{1}{3}}(1+x^{\frac{1}{2}})}$ (ii) $\int_0^1 \frac{dx}{\sqrt{x}(1+x)}$ $4 + 3 + 3$

3. (a) Show that $\int_0^{\pi/2} \sqrt{tan\theta} d\theta = 2\int_0^\infty \frac{x^2}{1+x^4} dx$.
(b) Express $\int_0^1 \frac{x}{\sqrt{1-x^5}} dx$ in terms of Beta function and then evaluate. $5 + 5$

4. (a) Evaluate $\int_0^\infty e^{-4x} x^{3/2} dx$
(b) Find the area bounded by the curves $y^2 = x^3$ and $x^2 = y^3$. $4 + 6$

5. (a) Find the entire area enclosed by the curve $r^2 = a^2 cos2\theta$.

(b) Find the approximate value of $\int_0^1 \frac{1}{\sqrt{1-x^2}} dx$ by Simpson's $\frac{1}{3}$ Rule taking 10 intervals correct upto five decimal places. $4 + 6$

6. (a) Show that the area of the loop formed by the curve $x^3 + y^3 = 3axy$ is $3\frac{a^2}{2}$.

(b) Find the length of the loop of the curve $9ay^2 = (x-2a)(x-5a)^2$. 5 + 5

7. (a) Find the volume of the solid generated by revolving one arc of the cycloid $x = a(\theta - sin\theta)$, $y = a(1 - cos\theta)$ about its base.

(b) Find the surface of the solid generated by revolution of the cardioide $r = a(1 + cos\theta)$.about the initial line. 5 +5

8. (a)Find the approximate value of $\int_0^1 e^x dx$ by Trapezoidal rule, taking five sub intervals correct upto five decimal places.
(b) Discuss the convergence of Gamms function
$\Gamma(n) = \int_0^\infty e^{-x}x^{n-1}dx$ 5 + 5