Ex/ME(M2)/BS/B/Math/T/221/2025

BACHELOR OF MECHANICAL ENGINEERING EXAMINATION, 2025

(2nd Year, 2nd Semester)

MATHEMATICS IV

Time : Three hours Full Marks : 100

Answer any ten questions

(Symbols/Notations have their usual meanings)

Answer any ten questions

Q.1 Compute the standard deviation from the following distribution of marks obtained by 90 students:

Marks	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
No.of Students	5	12	15	20	18	10	6	4

10

Q.2 Find the correlation coefficient from the following data:

x	10	12	13	16	17	20	25
y	19	22	24	27	29	33	37

10

Q.3 Find the regression of x on y from the following data

$$\sum_0^4 x = 24, \quad \sum_0^4 y = 44, \quad \sum_0^4 xy = 306, \quad \sum_0^4 x^2 = 164, \quad \sum_0^4 y^2 = 574$$

Find the value of x when $y = 6$. 10

Q.4 a) Define Distribution function. Show that distribution function is a monotonically non-decreasing function. 5

Q.b) If the random variable X has probability density function defined by

$$f(x) = \frac{1}{4}, \ -2 \leq x \leq 2$$
$$= 0 \quad elsewhere.$$

Obtain $P\{(2x+3) > 5\}$. 5

Q.5 Define Poisson distribution. Find the mean and variance of Poisson distribution. 10

Q.6a If A and B are mutually exclusive events and $P(A \cup B) \neq 0$ then prove that,

$$P\left(\frac{A}{A \cup B}\right) = \frac{P(A)}{P(A \cup B)}$$ 5

Q.b The probability of solving a problem by three students A, B and C are $\frac{2}{7}, \frac{3}{8}$ and $\frac{1}{2}$ respectively. If all of them try independently, find the probability that the problem is solved. Find also the probability that the problem could not be solved. 5

Q.7 Three identical boxes contain red and white balls. The first box contains 3 red and 2 white, the second box 4 red and 5 white, and the third box 2 red and 4 white balls. A box is chosen at random and a ball is drawn from it. If the ball drawn is red, what is the probability that the second box is chosen. 10

Q.8 Out of (2n+1) tickets consecutively numbered, three are drawn at random. Find the chance that the numbers on them are in A.P. 10

Q.9 a If $Z^{-1}[\bar{u}(z)] = u_n$ and $Z^{-1}[\bar{v}(z)] = v_n$ then show that

$$Z^{-1}[\bar{u}(z).\bar{v}(z)] = \sum_{m=0}^{n} u_m \, v_{n-m}$$ 5

b. Solve $u_{k+1} + u_k = 1$ if $u_0 = 0$. 5

Q.10 State Dirichlet's conditions for convergence of a Fourier series.

Find the Fourier series to represent the function $f(x)$ given by

$$f(x) = x \ for \ 0 \leq x \leq \pi$$
$$= 2\pi - x \quad for \ \pi \leq x \leq 2\pi$$

Hence deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} \dots \dots \dots = \frac{\pi^2}{8}$$ 10

Q.11 A periodic function of period 4 is defined as

$$f(x) = |x|, \ -2 \leq x \leq 2$$

Find its Fourier series expansion. 10

Q.12a. If $F(s)$ is the Fourier transform of $f(x)$ then $F\{f(ax)\} = \frac{1}{a}F({}^{s}/_{a})$ 5

b. Show that $F[f'(x)] = -isF(s)$ if $f(x) \to 0$ as $x \to \pm\infty$ 5

Q.13 a) If $L[f(t)] = F(p)$ then show that $L[e^{at}f(t)] = F(p-a)$ 5

b) Find $L[e^{-2t}\sin 3t]$ 5

Q.14 a) If $L[f(t)] = F(p)$ then show that $L\int_0^t f(u)du = \frac{1}{p}F(p)$ 5

b) Find Laplace inverse of $\frac{1}{p(p^2+a^2)}$ 5

Q.15 Use Laplace transform find the solution of

$$\frac{d^2y}{dt^2} + 25y = 10\cos 5t$$

Given $y(0) = 2$ and $y'(0) = 0$ 10