Ref. No. : Ex/ME(M2)/BS/B/MATH/T/211/**2025**

B.E. MECHANICAL ENGINEERING EXAMINATION 2025

Second Year First Semester

Mathematics -III

Full Marks -100 Time : 3 hr

(50 Marks for each Part)

Use Separate Answer scripts for each group.

Answer any five questions.

PART I (50 Marks)

Answer Question No 1 and any four from the followings.

1. What is the necessary condition for a first order first degree differential equation to be exact? 2

2. Solve the following differential equations (any two):
(i) $(x^4 + y^4)dx - xy^3dy = 0$
(ii) Show that the equation $(x^3 - 3x^2y + 2xy^2)dx - (x^3 - 2x^2y + y^3)dy = 0$ is exact and find the solution.
iii) $y = 2px + p^3y^2$, where $p = \frac{dy}{dx}$ [CO4] 6 + 6

3. (i) Solve the differential equation $\frac{d^2y}{dx^2} + 4y = 4tan\ 2x$ by method of variation of parameter.
(ii) $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + 6y = log\ x^2$ [CO4] 6 +6

4. Show that x = 0 is a regular singular point of $(2x + x^3)\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6xy = 0$ and find the series solution of the equation near the point $x = 0$. [CO4] 2+10

5. (i) Find the general solution of the differential equation
$\frac{d^2y}{dx^3} - 2\frac{dy}{dx} + y = x^2e(3x)$ [CO4]
(ii) Solve the differential equation
$\frac{y-z}{yz}p + \frac{z-x}{zx}q = \frac{x-y}{xy}$ [CO6] 6 +6

6. Solve the following differential equations where $p = \partial z/\partial x, q = \partial z/\partial y$:
(i) $z(x + y)p + z(x - y)q = x^2 + y^2$
(ii) $y^2p - xyq = x(z - 2y)$ [CO6] 6 +6

7. A string is stretched and fastened to two points $x = 0$ and $x = l$ apart. Motion is started by displacing the string into the form $y = f(x)$ from which it is released at time t = 0. Derive the governing partial differential equation and state initial and boundary conditions. Find the displacement of any point on the string at a distance of x from one end at time t. [CO5, CO6] 4+8

[Turn over

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PART II (50 Marks)

1 (i) Determine the subspace of $\mathbb{R}^3$ spanned by the vectors $\alpha = (1,2,3)$ and $\beta = (3,1,0)$. Examine by

(a) $\gamma = (2,1,3)$ is in the subspace.

(b) $\delta = (-1,3,6)$ is in the subspace.

(ii) Prove that the set $S = \{(1,1,0),(1,0,1),(0,1,1)\}$ is a basis of $\mathbb{R}^{\#}$ [CO1] 6+4

2 (i) Intersection of two subspaces of a vector space V over a field F is a subspace of V.

(ii) For what value of k does the set $S = \{(k,1,1),(1,k,1),(1,1,k)\}$ form a basis of $\mathbb{R}^3$

(iii) Prove the vectors $\{(1,2,2),(2,1,2),(2,2,1)\}$ is linearly independent in R^3 [CO2] 3+4+3

3 (i) Find rational canonical form corresponding to the matrix $A = \begin{bmatrix} 0 & -4 & 85 \\ 1 & 4 & -30 \\ 0 & 0 & 3 \end{bmatrix}$

(i) Find the Jordan canonical form J of $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ [CO1] 5+5

4 (i) Let A be a 5×5 matrix with characteristics polynomial $(x-1)^3(x-2)^2$ and minimal polynomial $(x-1)^2(x-2)$. What will be the possible Jordan canonical for of A .

(ii) Cosider the matrices $A = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 2 & -1 \\ 0 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ then prove that B is the Jordan canonical form of A [CO1] 5+5

5 (i) Suppose the mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x, y) = (x + y, x)$. Show that T is linear operator.

(i) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear operator defined by $T(x, y) = (2x + 3y, 4x - 5y)$. Find the matrix representation of T relative to the basis $S = \{u_1, u_2\} = \{(1, 2), (2, 3)$. [CO3] 5+5

6 (i) Find the eigen value of the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}$.

(ii) Find the minimal polynmial $m(t)$ of the matrix $B = \begin{bmatrix} 2 & 2 & -5 \\ 3 & 7 & -15 \\ 1 & 2 & -4 \end{bmatrix}$.

[CO2] 5+5

7 (i) State and prove Cauchy-Schwartz inequality.

(ii) If v_1 and v_2 are eigenvectors of a normal operator T on an inner product space V with respect to eigenvalues λ_1 and λ_2 respectively, then prove that v_1 and v_2 are orthogonal.

[CO1] 7+3