MASTER OF SCIENCE EXAMINATION, 2025

(2^{nd} Year, 1^{st} Semester)

MATHEMATICS

PAPER : CORE 11

(Integral Equation, Integral Transform and Calculus of Variation)

Time : *Two Hours* | *Full Marks* : 40

Symbols/Notations have their usual meaning
Use separate Answer-Scripts for each Part

PART—I

(24 Marks)

Answer Question No.1 and *any* **two** from the rest :

1. *(a)* Derive Euler's equation in the form $\frac{\partial f}{\partial y}-\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)=0$ 4

(b) Answer *any* **one** :

(i) Find a curve in the *x–y* plane along which particle that starts from a point $P(x_1,y_1)$ arrives at another point $Q(x_2,y_2)$ in shortest time. 4

(ii) Find the extremals of the functional $J[y,z]=\int_0^{\pi/2}\left(y'^2+z'^2+2yz\right)dx$, subject to the boundary conditions $y(0)=0,\ y(\pi/2)=1,$ $z(0)=0,\ z(\pi/2)=1$. 4

2. *(a)* Obtain the nontrivial solution of $\phi(x)-\lambda\int_0^{2\pi}\sin(x+t)\phi(t)dt=0$. 4

(b) Use Hilbert Schmidt theorem to solve the following integral equation $\phi(x)-\int_0^{2\pi}\sin(x+t)\phi(t)dt=x$, $0\le x\le 2\pi$. 4

3. *(a)* For an integral operator $Ku=\int_b^a k(x,t)u(t)dt$; prove that $(K\phi,\psi)=(\phi,K^*\psi)$. 3

(b) Find the n^{th} iterated kernels of the following integral equation

$$\phi(x)=\left(e^x-\frac{e}{2}+\frac{1}{2}\right)+\frac{1}{2}\int_0^1\phi(t)dt,\quad 0\le x\le 1$$ 1

(c) Obtain Neumann series solution of

$$u(x)=x+\lambda\int_0^1 xt\,u(t)dt,\ 0\le x\le 1$$ 4

4. *(a)* For what values of λ, does the unique solution of $\phi(x)=x+\lambda\int_0^1(1+x+t)\phi(t)dt,\ 0\le x\le 1$ exist? 3

(b) Obtain an integral equation for $y''(x)+\lambda y(x)=0$ $0\le x\le 1$; $ay(0)=by(1)$, $by'(0)=ay'(1)$ where a, b are constants. 5

PART—II

(16 Marks)

(Integral Transforms)

Answer *any* **two** questions :

1. *(a)* State and prove general Parseval relation for Fourier transform. 4

(b) Find $\mathcal{H}_0\left[\frac{e^{-ar}}{r}\right]$, where $a>0$. 4

2. *(a)* If $\mathcal{H}_n[f(r)]=F_n(\rho)$, then show that

$$\mathcal{H}_n\left[\frac{f(r)}{r}\right]=\frac{\rho}{2n}[F_{n-1}(\rho)+F_{n+1}(\rho)].$$ 2

(b) Use Fourier transform to solve the differential equation $\frac{d^2f}{dx^2}-f=e^{-|x|}$, $-\infty<x<\infty$ with f, $f'\to 0$ as $|x|\to\infty$. 6

3. Find $\mathcal{L}^{-1}\left\{\frac{p}{p^2+a^2}\right\}$ by using the complex integration formula. 8

★★★