Master Of Science Examination, 2025

(2nd Year, 1st Semester)

MATHEMATICS

Paper : DSE/04/A15

(Theory of Semigroups – 1)

Time : *Two Hours* *Full Marks* : 40

Symbols/Notations have their usual meanings.

*Answer any **four** questions.*

1. *(a)* Define rectangular band and semilattice. When is a rectangular band a semilattice? Explain with an example.

   (b) Define a periodic semigroup. Show that in a periodic semigroup every element has a power which is idempotent. Explain with an example of such semigroup.

   5+5=10

2. *(a)* Define a congruence ρ on a semigroup S. If $\phi : S \to T$ is a homomorphism such that $\rho \subseteq \ker \phi$, then show that there is a unique homomorphism $\beta : S/\rho \to T$ such that $\mathrm{ran}(\beta) = \mathrm{ran}\ (\phi)$ and $\phi\beta = \rho^k$, when ρ^k is the natural epimorphism from S onto S/ρ.

   (b) Let S be a semigroup. Define $E(S)$. Give two examples of semigroups S and T such that $E(S) = \phi$ and $E(T) = T$.

   (1+5)+(1+3)=10

3. *(a)* Define the Green's Equivalence relations L and R on a semigroup. Prove that $L \circ R = R \circ L$.

(b) Define the Green's equivalence relations D and H on a semigroup. If a and b are D-equivalent element of S, then show that $| H_a | = | H_b |$, where H_x denotes the equivalence class of $x \in S$ under H. 4+6=10

4. *(a)* Define a regular semigroup. Let e, f be idempotents in a regular semigroup S. Then show that $(e, f) \in D$ if and only if there exists an element $a \in S$ and an inverse a' of a such that $aa' = e$, $a'a = f$.

(b) Let ρ be a congruence on a regular semigroup S. If $a\rho$ is an idempotent in S/ρ, then show that there exists an idempotent e in S such that $a\rho = e\rho$. 6+4=10

5. *(a)* Show that a semigroup S is completely regular if and only if S is union of groups.

(b) Let e and f be idempotents in a regular semigroup S. Then prove that $S(e, f)$ is a sub semigroup of S and also a rectangular band, where $S(e, f) = \{g \in E : ge = fg = g, egf = ef\}$ is a sandwich set of e and f and E is the set of all idempotents of S. 6+4=10

6. *(a)* Define an orthodox semigroup. Let S be a regular semigroup such that if e is an idempotent, then every inverse of e is idempotent. Prove that S is an orthodox semigroup.

(b) In an orthodox semigroup, e is an idempotent and $a \in S$, then show that for every inverse a' of a, the elements $a'ea$ and aea' are both idempotent. 6+4=10

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