

Ex/SC/MATH/PG/DSE/TH/04/A25/2025

MASTER OF SCIENCE EXAMINATION, 2025

(2nd Year, 1st Semester)

MATHEMATICS

PAPER : DSE-04/A25

(Modelling of Biological Events)

Time : Two Hours

Full Marks : 40

Symbols/Notations have their usual meaning.

Use separate Answer-Scripts for each Part.

The figures in the margin indicate full marks.

PART—I

(24 Marks)

Answer **Question No.1** and *any two* from the rest.

1. Solve the delay differential equation

$$\frac{dN(t)}{dt} = N(t-1), \quad t \geq 0$$

with the history function $N(t) = 1 \quad \forall t \in [-1, 0]$. 4

2. Suppose the population densities of a prey and its predators at time t are x and y , respectively. If prey grows logistically in the absence of predators and the predator's functional response is type I, then construct a predator-prey model with a linear death of predators in the absence of its prey. If the predator has two life stages, viz. immature and mature and only matured predators have the predation and reproduction

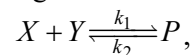
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(2)

capabilities and τ is the maturation delay, then reconstruct your predator-prey model with the necessary assumptions. Find the stability criteria of the predator-free equilibrium of your proposed age-structured model. $2+3+5=10$

3. Consider the following chemical reaction :



where k_1, k_2 are the rate constants.

- (a) Use the mass action law to construct the differential equations for the rate equations of the chemicals X , Y and P .
- (b) What is the equilibrium constant of the reaction and what is its significance?
- (c) If Y is replaced by X then describe the new rate equations. Prove in this case that the rate of production of P is half of the rate of change in X . What quantity is conserved in this reaction? $2+2+6=10$

4. The Lotka-Volterra predator-prey model with diffusion is represented by

$$\frac{\partial F}{\partial t} = D_1 \frac{\partial^2 F}{\partial x^2} + \alpha F - \beta FG, \quad \frac{\partial G}{\partial t} = D_2 \frac{\partial^2 G}{\partial x^2} + \delta FG - \gamma G,$$

where $F(x, t)$ and $G(x, t)$ are the prey and predator densities at position x at time t . All parameters are positive and carry the usual meanings. Considering positive initial values and zero-flux boundary conditions, discuss the stability and diffusion-driven instability (if any) of the homogeneous coexisting equilibrium point. 10

(3)

PART—II (16 Marks)

Answer *any two* questions from the following :

1. Write down a discrete mathematical models of two mutualistic species. Determine the steady states and discuss the stability properties of the interior equilibrium. 8
2. Write down a discrete simple three species food chain model. Determine the steady states and discuss the stability properties of the prey and intermediate predator equilibrium. 8
3. Describe Yule-Furry model (pure birth process) of stochastic epidemic model. Also find the variance in the number of infective at any time t . 8

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