Master Of Science Examination, 2025

(2nd Year, 1st Semester)

MATHEMATICS

Paper : DSE-04/A25

(Modelling of Biological Events)

Time : *Two Hours* *Full Marks* : 40

Symbols/Notations have their usual meaning.
Use separate Answer-Scripts for each Part.

The figures in the margin indicate full marks.

PART—I

(24 Marks)

Answer **Question No.1** and *any* **two** from the rest.

1. Solve the delay differential equation

$$\frac{dN(t)}{dt} = N(t-1), \quad t \geq 0$$

with the history function $N(t) = 1 \; \forall t \in [-1, 0]$. 4

2. Suppose the population densities of a prey and its predators at time t are x and y, respectively. If prey grows logistically in the absence of predators and the predator's functional response is type I, then construct a predator-prey model with a linear death of predators in the absence of its prey. If the predator has two life stages, viz. immature and mature and only matured predators have the predation and reproduction

capabilities and τ is the maturation delay, then reconstruct your predator-prey model with the necessary assumptions. Find the stability criteria of the predator-free equilibrium of your proposed age-structured model. 2+3+5=10

3. Consider the following chemical reaction :

$$X + Y \underset{k_2}{\overset{k_1}{\rightleftharpoons}} P,$$

where k_1, k_2 are the rate constants.

(a) Use the mass action law to construct the differential equations for the rate equations of the chemicals X, Y and P.

(b) What is the equilibrium constant of the reaction and what is its significance?

(c) If Y is replaced by X then describe the new rate equations. Prove in this case that the rate of production of P is half of the rate of change in X. What quantity is conserved in this reaction? 2+2+6=10

4. The Lotka-Volterra predator-prey model with diffusion is represented by

$$\frac{\partial F}{\partial t} = D_1 \frac{\partial^2 F}{\partial x^2} + \alpha F - \beta FG, \quad \frac{\partial G}{\partial t} = D_2 \frac{\partial^2 G}{\partial x^2} + \delta FG - \gamma G,$$

where $F(x,t)$ and $G(x,t)$ are the prey and predator densities at position x at time t. All parameters are positive and carry the usual meanings. Considering positive initial values and zero-flux boundary conditions, discuss the stability and diffusion-driven instability (if any) of the homogeneous coexisting equilibrium point. 10

PART—II (16 Marks)

Answer *any* **two** questions from the following :

1. Write down a disctere mathematical models of two mutualistic species. Determine the steady states and discuss the stability properties of the interior equilibrium. 8

2. Write down a discrete simple three species food chain model. Determine the steady states and discuss the stability properties of the prey and intermediate predator equilibrium. 8

3. Describe Yule-Furry model (pure birth process) of stochastic epidemic model. Also find the variance in the number of infective at any time t. 8

★ ★ ★