Master Of Science Examination, 2025

(2^{nd} Year, 1^{st} Semester)

MATHEMATICS

Paper : DSE/04/A26

(Probability, Games and Queuing Theory)

Time : *Two Hours* | *Full Marks* : 40

Symbols/Notations have their usual meanings.

Use seperate Answer scripts for each part.

PART—I

Answer *any* **two** questions, each question carries **ten** marks.

1. *(a)* Define a field, a ring, a sigma-ring and a sigma-field.

(b) Give an example of a sigma-ring which is not a sigma field.

(c) Give an example of a field which is not a sigma-field.

4+3+3=10

2. *(a)* Prove that the characteristic function of any random variable is uniformly continuous over the set of real numbers.

(b) Find the characteristic function of the random variable X which has density $f(x)$ given below :

$$f(x) = \frac{1}{\pi} \cdot \frac{1 - \cos x}{x^2},\ x \in \mathbb{R}$$

2+8=10

3. State and prove the Uniqueness and Inversion theorem on characteristic functions. 10

PART—II

Answer *any* **two** questions, each question carries **ten** marks.

1. (a) Define the minimax-maximin principle and saddle point of game theory.

   (b) Let $f(x,y)$ be a real valued function of x and y for $x \in A$ and $y \in B$; A, B being two sets. Suppose that both $\max_{x\in A} \min_{y\in B} f(x,y)$ and $\min_{y\in B} \max_{x\in A} f(x,y)$ exist, then the necessary and sufficient condition that $f(x,y)$ possesses a saddle point is

   $$\max_{x\in A} \min_{y\in B} f(x, y) = \min_{y\in B} \max_{x\in A} f(x, y).$$

   2+8=10

2. (a) Two companies A and B are competing for the same product. Their different strategies are given in the following payoff matrix. Use linear programming to determine the best strategies for both the players.

		Player B		
		B_1	B_2	B_3
Player A	A_1	3	−2	4
	A_2	−1	4	2

   (b) Players A and B play a game as follows :

   They simultaneously and independently write one of the three numbers 1, 2, 3. If the sum of the numbers written be even, then B pays to A this sum in rupees. If it be odd, then A pays the sum to B in rupees. Derive the payoff matrix of the game for A and solve it. 5+5=10

3. (a) Derive the differential-difference equations for the queuing model $(M / M / 1) : (\infty / FCFS)$. Obtain the steady state solution of the model and also find the expected value of the queue length.

   (b) A barber with one man shop takes exactly 25 minutes to complete one hair cut. If customers arrive in a Poisson fashion at an average rate of one ever 40 minutes, how long on the average must a customer wait for service? 7+3=10

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