Master Of Science Examination, 2025

(2nd Year, 1st Semester)

MATHEMATICS

Paper : DSE 02A

[Numerical Analysis I (Theory)]

Time : 1 *Hour* 15 *Minutes* *Full Marks* : 24

(Use separate answer script for each Part)

(Symbols and Notations have their usual meanings)

PART—I

(16 Marks)

Answer *any* **two** questions

1. *(a)* Consider the linear system $\boldsymbol{Ax}=\boldsymbol{b}$, where $\boldsymbol{A}$ is an $n\times n$ matrix. Show that the iteration method, defined as

$$\boldsymbol{x}^{(k+1)}=\boldsymbol{Hx}^{(k)}+\boldsymbol{c},\ \ k=0,1,2,\ldots$$

converges to the exact solution of $\boldsymbol{Ax}=\boldsymbol{b}$, for any initial vector $\boldsymbol{x}^{(0)}$ if $\|\boldsymbol{H}\|<1$, where $\boldsymbol{H}$ is the iteration matrix and $\boldsymbol{c}$ is a column vector.

(b) Deduce Thomas algorithm for the solution of tri-diagonal system of linear equations. 3+5

2. *(a)* Find the Hermite interpolating polynomial from the triplet (x_i, y_i, y_i') of values of (–1,1,–5), (0,1,1), (1,3,7) and hence find the value of $f(0.5)$.

(b) Distinguish between interpolation and best approximation. 6+2

3. (a) Define the condition number of a matrix.

(b) Use Cholesky method to solve the following system of equations :

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 5 \\ 6 \\ -10 \end{bmatrix}$$

2+6

PART—II

(8 Marks)

Answer *any* **one** question

4. Define Gerschgorin's Disk. Let R_k be the sum of the moduli of the elements along the kth row excluding the diagonal element a_{kk}. Then prove that every eigenvalue λ of a square matrix A of order n lies inside or on the boundary of at least one of the circles

$$|\lambda - a_{kk}| = R_k = \sum_{j=1,\ j\neq k}^{n} |a_{jk}|,\ \ k = 1, 2, \ldots, n$$

2+6

5. (a) Write an algorithm to determine all the eigenvalues and the corresponding eigenvectors of a real symmetric matrix of order n using the Jacobi's method.

(b) Estimate the eigenvalues of the following matrix using Gerschgorin bounds :

$$\begin{pmatrix} 2 & 1 & 0 \\ 1 & 3 & -1 \\ 1 & 0 & -2 \end{pmatrix}$$

5+3

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