Master Of Science Examination, 2025

(2nd Year, 1st Semester)

MATHEMATICS

Paper : CORE/10

(Dynamical System)

Time : *Two Hours* *Full Marks* : 40

Symbols/Notations have their usual meaning.

Use separate Answer-Scripts for each Group.

The figures in the margin indicate full marks.

GROUP—A

(20 Marks)

Answer *any* **two** questions.

1. *(a)* Find Hamiltonian function for the system

$$\dot{x} = y$$
$$\dot{y} = -x^3 + x$$

(b) Draw the bifurcation diagram in $\tau - \delta$ plane for 2D linear autonomous system $\dot{X} = AX$, $X \in R^2$, $A = (a_{ij})_{2\times 2}$ with $\tau = trace\, A$, $\delta = det(A)$ and justify your answer.

(c) If (0, 0) is a focus of the Hamiltonian system in R^2 then show that (0, 0) is not a strict local optimum of the Hamiltonian function. 2+5+3=10

2. *(a)* State fundamental theorem for linear system in R^n.

(b) Find the evolution operator $\phi^t : R^2 \to R^2$ for the system $\dot{x} = f(x)$ with $f(x) = \begin{pmatrix} -x_1 \\ x_2 + x_1^2 \end{pmatrix}$. Verify the properties of flow ϕ^t and hence show that the set $S = \{(x_1, x_2) \in R^3;\ x_2 = -x_1^2/3\}$ is invariant under the flow ϕ^t.

(c) Characterise the bifurcation with respect to the parameter $\beta \in R$ for the system

$$\dot{x} = \beta + x^2$$
$$\dot{y} = -y$$

Draw the phase portrait. 1+5+4=10

3. *(a)* What do you mean by Heteroclinic bifurcation?

(b) State stable manifold theorem for the system $\dot{x} = f(x)$, $x \in R^n$.

(c) Determine the nature of the equilibrium point (0, 0) of the system

$$\dot{x} = y^3$$
$$\dot{y} = -x^3$$

(d) Show that the system

$$\dot{x} = y + \frac{x}{4}(1 - 2r^2)$$
$$\dot{y} = -x + \frac{y}{2}(1 - r^2)$$

where $r^2 = x^2 + y^2$, has at least one periodic orbit. Draw the phase portrait. 2+2+2+4=10

GROUP—B
(20 Marks)

Answer *any* **four** questions.

1. *(a)* Find the order of the difference equation
$\Delta^3 x(n) + \Delta^2 x(n) + \Delta x(n) - x(n) = 0.$

(b) Solve the equation by the annihilator method
$x(n+2) - 2x(n+1) + x(n) = 3n + 5.$ 1+4=5

2. Find the general solution of the system of linear equations

$$\begin{pmatrix} x(n+1) \\ y(n+1) \end{pmatrix} = \begin{pmatrix} -2 & -1 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} x(n) \\ y(n) \end{pmatrix}$$

Sketch the phase portrait for various initial conditions. 5

3. Find all period two points of the map $G(x) = 4x(1-x)$, $x \in [0, 1]$. Show that they are sources. 5

4. Suppose (p, q) is a fixed point of the system of equations

$$x_{n+1} = f(x_n, y_n)$$
$$y_{n+1} = g(x_n, y_n)$$

Derive the condition for which (p, q) is asymptotically stable. When does your analysis fail? 5

5. Define bifurcation in a one parameter family of maps. Discuss bifurcation in the map $f(x) = rx - x^3$ near $r = 1$. Draw the bifurcation diagram. 5

6. Prove that the Bernoulli shift map given by $f(x) = 2x$ (mod 1), $x \in [0, 1]$ shows chaotic orbits. 5

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