Master Of Science Examination, 2025

(2nd Year, 1st Semester)

MATHEMATICS

Paper : DSE-03/A4

(Graph Theory – I)

Time : *Two Hours* *Full Marks* : 40

Symbols/Notations have their usual meanings

All questions carry equal marks

Answer *any* **four** questions : 10×4=40

1. (a) Describe and draw the Petersen graph. Show that the Petersen graph is a triangle-free 3-regular graph.

   (b) Define the *diameter* of a graph. Let G be a simple connected graph. If diameter of G is greater than or equal to 3, then show that the diameter of $\overline{G}$, the complement of G, is less than or equal to 3.

2. (a) Define a *component* of a graph. Let G be a graph with n vertices and e edges. Then prove that G has at least $n-e$ components.

   (b) Define a *cut edge* of a graph. Let $G=(V, E)$ be a connected graph and $e \in E$. Then show that e is a cut edge if and only if e does not belong to any cycle of G.

3. (a) Define a *Hamiltonian cycle*. If $G=(V, E)$ has a Hamiltonian cycle, then for each nonempty set $S \subseteq V$, the graph $G-S$ has at most |S| components. Give an example of a graph satisfying the above condition but does not have a Hamiltonian cycle.

   (b) What is a *matching* in a graph? State and prove Hall's theorem for matching in a bipartite graph.

4. (a) Define a *bipartite graph* and an *Eulerian graph*. Let G be an Eulerian bipartite graph. Show that G has even number of edges.

   (b) Define a *vertex cover* of a graph G. Prove that in a bipartite graph G, the maximum size of a matching G is equal to the minimum size of a vertex cover of G.

5. (a) Define the *chromatic number* $\chi(G)$ of a graph G. Define the cartesian product $G \square H$ of two graphs G and H. Prove that $\chi(G \square H)=\max\{\chi(G), \chi(H)\}$.

   (b) Define the *chromatic polynomial* of a graph. Find the chromatic polynomial of a tree with n vertices.

6. (a) Define a *chordal graph*. Show that a graph G is chordal if and only if it has a perfect elimination ordering.

   (b) Define an *interval graph* and a *perfect graph*. Show that an interval graph is perfect.

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