MASTER OF SCIENCE EXAMINATION, 2025

(2nd Year, 1st Semester)

MATHEMATICS

PAPER : DSE/03/A7

(Nonlinear and Dynamic Programming)

Time : *Two Hours* *Full Marks* : 40

The figures in the margin indicate full marks.

Symbols/Notations have their usual meanings.

Use separate answer script for each Part.

Part—I (20 Marks)

*Answer any **two** questions.*

1. *(a)* Let S be a non-empty open convex subset of R^n and $f : S \to R$ be twice differentiable on S. Then show that f is a convex function on S iff the Hessian matrix $\nabla^2 f(x)$ is positive semi-definite $\forall x \in S$.

(b) Use Lagrange multiplier method to solve the optimization problem :

Minimize $f(x, y) = \frac{x^3}{3} - \frac{3y^2}{2} + 2x$

subject to $x - y = 0$. 5+5=10

2. *(a)* Let $S \subseteq R^n$ be a convex set and $f : S \to R$. Then show that f is a concave function on S iff its hypograph H_f is a convex set.

(b) Using Steepest Descent method, minimize $f(x_1, x_2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ starting from the point $X_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Perform two iterations. 5+5=10

3. *(a)* Show that $f(x) = x^T Qx + c^T x$, where $c, x \in R^n$, is a convex function if Q is a positive semi-definite symmetric matrix.

(b) Distinguish between interior penalty function and exterior penalty function methods. Minimize $f(x_1, x_2) = x_1^2 + x_2^2$ subject to $1 - x_1 - x_2 \leq 0$, using interior penalty function method with the calculus method of unconstrained minimization. 3+7=10

Part—II (20 Marks)

*Answer any **two** questions.*

1. *(a)* Define the Fibonacci sequence $\{f_n\}$.

(b) Find the ratios *(i)* f_{12} / f_{11} and *(ii)* $\frac{f_{22}}{f_{21}}$ using Dynamic Programming Algorithm.

(c) As $n \to +\infty$, guess the limit of $\frac{f_{nH}}{f_n}$, what is the name of this limit? 2+6+2=10

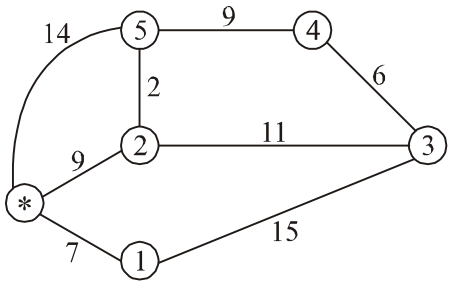
2. Using Dynamic Programming Method, solve the following (0, 1) Knapsack problem :

Item	Weight (Kgs)	Value
I_0	3	100
I_1	8	40
I_2	9	90
I_3	8	110

Capacity of the Knapsack is 20 kgs.

Note : All the tables are to be displayed with justifications. 10

3. Consider the following undirected graph with single source ⊛. Find the shortest path from ⊛ to all the other nodes.



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