Master Of Science Examination, 2025

(2nd Year, 1st Semester)

MATHEMATICS

Paper : DSE/03/A10

(Advanced Differential Geometry)

Time : *Two Hours* *Full Marks* : 40

Symbols/Notations have their usual meanings.

*Answer any **four** questions.*

[CO1]

1. *(a)* Define 1-parameter group of transformations on a differentiable manifold M. If $M = \mathbb{R}^2$ and $\phi : \mathbb{R} \times M \to M$ is defined by $\phi(t, (x, y)) = (xe^{2t}, ye^{-2t})$, then check whether ϕ is an 1-parameter group of transformation or not. If possible find its generator. 2+4

(b) If $w = xydx + \left(\frac{1}{2}x^2 - y\right)dy$ is an 1-form on $\mathbb{R}^2$, check whether w is closed or not. 4

[CO2]

2. *(a)* Prove that in a Riemannian manifold Riemannian metric is covariantly constant. 4

(b) Prove in terms of local coordinates the components of the Riemannian connection Γ^i_{jk} are given by

$$g_{im}\Gamma^i_{jk} = \frac{1}{2}\left(\frac{\partial g_{mk}}{\partial x^j} + \frac{\partial g_{jm}}{\partial x^k} - \frac{\partial g_{jk}}{\partial x^m}\right).$$ 6

[CO3]

3. (a) Describe the concept of sectional curvature on a Riemannian manifold. When is a manifold called manifold of constant curvature? Explain with an example. 4

(b) Prove that $g(R(X,Y)Z,U) = -g(R(X,Y)U,Z)$. Where R is the Riemann curvature tensor on a Riemannian manifold M. 6

[CO3 & CO4]

4. (a) Define Weyl conformal curvature tensor C on a Riemannian manifold of dimension n. Hence obtain Goldberg's expression of C. 2+3

(b) Prove that projectively flat Riemannian manifold is an Einstein manifold. 5

[CO4 & CO1]

5. (a) Prove that if a vector field V on an almost complex manifold M is strictly contravariant almost analytic then Nijenhuis tensor $N(V,X)$ vanishes for every vector field X on M. What is the geometric significance of the result? 5

(b) Prove that in a Kähler manifold the Ricci tensor S satisfies :

(i) $S(\overline{X},\overline{Y}) = S(X,Y)$

(ii) $QX + \overline{Q\overline{X}} = 0$

where $g(QX, Y) = S(X, Y)$ and $F(X) = \overline{X}$. 3+2

[CO1]

6. (a) Prove that for an almost contact manifold $\left(M^{2m+1},\phi,\xi,\eta\right)$, (i) $\phi\circ\xi = 0$, (ii) $\eta\circ\phi = 0$ and (iii) rankϕ = 2m. 2+2+2

(b) Give an example of a 5-dimensional almost contact manifold with justification. 4

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