2. Evaluate

$$\int_0^1 x^3(1-x^2)^{\frac{5}{2}}\,dx$$ **[CO1]**

3. Evaluate

$$\int_0^1 \frac{dx}{(1-x^6)^{1/6}}\,dx$$ **[CO2]**

4. Find the area bounded by the curves :

$$y = x^2 \text{ and } x = y^2$$ **[CO3]**

5. Show that the necessary and sufficient condition for a vector function $\vec{F}(t)$ to have constant magnitude is

$$\vec{F}(t) \bullet \frac{d\vec{F}(t)}{dt} = 0$$ **[CO4]**

6. Find the constants a, b so that the surfaces

$$ax^2 - byz = (a+2)x$$

will be orthogonal to the surface

$$4x^2y + z^3 = 4 \text{ at } (1, -1, 2).$$ **[CO2]**

7. Prove that

$$(i) \text{ curl grad } \phi = 0, \ (ii) \text{ div curl } \vec{F} = 0$$ **[CO4]**

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BACHELOR OF SCIENCE EXAMINATION, 2025

(1st Year, 1st Semester)

MATHEMATICS

PAPER : MINOR/TH/11/101A

(Mathematics—1)

Time : *Two Hours* *Full Marks* : 40

Use separate Answer-Scripts for each Group.

GROUP—A (20 Marks)

1. Answer *any* **three** questions : 2×3=6

[CO1]

(a) If the pair of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angles between the other pair, then show that $pq = -1$.

(b) What do you mean by Radical axis? Find the Radical axis of two circles given by $x^2 + y^2 + 4x - 2y + 9 = 0$ and $x^2 + y^2 + 2x + 3y - 5 = 0$.

(c) Find the equation of the sphere on the join of $P(2,-3,4)$ and $Q(-5,6,-7)$ as diameter.

(d) If $\log y = \tan^{-1} x$, then show that

$$(1+x^2)y_2 + (2x-1)y_1 = 0$$

(e) State Rolle's theorem. Is Rolle's theorem applicable for $f(x) = \tan x$ in $[0, \pi]$.

2. Answer *any* **two** questions : 3×2=6

[CO2]

(a) Find the coordinates of the limiting points of the co-axial system given by the circles

$x^2 + y^2 - 6x - 8y + 5 = 0$ and

$x^2 + y^2 - 8x - 10y + 5 = 0$.

(b) Find the equation of the plane passing through the points (2, 1, –1) and is perpendicular to each of the planes $x - y + z = 1$ and $3x + 4y - 2z = 0$.

(c) If $\lim_{x \to 0} \frac{(a \sin x - \sin 2x)}{(\tan^3 x)}$ is finite, find the value of a and the limit.

3. Answer *any* **one** question : 4×1=4

[CO3]

(a) In the Mean value Theorem $f(x+h) = f(x) + hf'(x+\theta h)$, show that if $f(x) = Ax^2 + Bx + C$, $A \neq 0$, then $\theta = \frac{1}{2}$. Give a geometrical interpretation of the result.

(b) Find the centre and radius of the circle given by $x^2 + y^2 + z^2 - 2y - 4z - 11 = 0$ and $x + 2y + 2z = 15$.

4. Answer *any* **one** question : 4×1=4

[CO4]

(a) Find the equation of the straight line in symmetrical form which is given by the general equation as $x + 5y - z - 7 = 0$ and $2x - 5y + 3z + 1 = 0$.

(b) If $u = \tan^{-1} \frac{(x^3 + y^3)}{(x - y)}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

GROUP—B (20 Marks)

Answer *any* **four** questions : 5×4=20

1. Find the value of the integral :

$$\int_0^\infty \frac{\tan^{-1} \alpha x}{x(1+x^2)} dx$$ **[CO1]**