*(b)* Check convergency of

$$\int_0^\infty \frac{1}{x}\left(\frac{1}{1+x} - e^{-x}\right)dx$$

*(c)* Find the smaller of the area (using double integration) bounded by $y = 2 - x$ and $x^2 + y^2 = 4$.

(3+3+3) **(CO4)**

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**Ex/ARTS/MATH/UG/MINOR/TH/11/101/2025**

**Bachelor of Arts Examination, 2025**

**( 1st Year, 1st Semester )**

**MATHEMATICS – I**

**Paper : MATH/UG/MINOR/TH/11/101**

*Time* : *Two Hours* *Full Marks* : 30

*The figures in the margin indicate full marks.*

*Symbols / Notations have their usual meanings.*

**1.** *(a)* Show that the semi-latus rectum of a conic is a harmonic mean between the segments of any focal chord.

*(b)* For the ellipse $2x^2 + 3y^2 - 24 = 0$, find pair of semi-conjugate diameters inclined at an angle $\tan^{-1} 7$.

(3+3) **(CO1)**

**( OR )**

**2.** *(a)* Show that the locus of the poles of chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ which subtended a right angle at the centre is $\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{a^2} + \frac{1}{b^2}$.

*(b)* Find the values of $c$ for which the plane $x + y + z = c$ touches the sphere $x^2 + y^2 + z^2 - 2x - 2y - 2z = 6$.

(3+3) **(CO1)**

**3.** *(a)* Prove that $r^n\vec{r}$ is irrotational for every $n$. Find the value of $n$ for which it is solenoidal.

*(b)* Evaluate by Stokes' theorem $\int_C (e^x dx + 2y dy - dz)$, where $C$ is the curve $x^2 + y^2 = 4$, $z = 2$.

(5+3) **(CO2)**

**( OR )**

**4.** *(a)* Evaluate by Green's theorem

$$\int_C \{(y - \sin x)\,dx + (\cos x)\,dy\},$$

where $C$ is the triangle bounded by $y = 0$, $x = \frac{\pi}{2}$, $y = \frac{2x}{\pi}$.

*(b)* Find the equation of normal line to the surface $xyz = 4$ at the point $(\vec{i} + 2\vec{j} + 2\vec{k})$.

(5+3) **(CO2)**

**5.** *(a)* If $u = \sin^{-1}\left\{\dfrac{x^{\frac{1}{3}} + y^{\frac{1}{3}}}{x^{\frac{1}{2}} + y^{\frac{1}{2}}}\right\}$, then using Euler theorem prove that

$$x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = \frac{\tan u}{144}(13 + \tan^2 u).$$

*(b)* Find $\lim\limits_{x \to 0}\left\{\dfrac{1}{x} - \cot x\right\}$.

(5+2) **(CO3)**





**( OR )**

**6.** *(a)* Using Mean-value theorem to prove

$$\frac{x}{1+x} < \ln(1+x) < x, \text{ if } x > 0.$$

*(b)* If $y = (\sin ax + \cos ax)$, then prove that

$$y_n = a^n\{1 + (-1)^n \sin 2ax\}^{\frac{1}{2}}.$$

(4+3) **(CO3)**

**7.** *(a)* Change the order of the integral $\int_0^1 \int_y^{3-y} f(x,y)\,dx\,dy$.

*(b)* Using properties of definite integral to prove

$$\int_0^{\frac{\pi}{2}} \frac{x}{\sin x + \cos x}\,dx = \frac{\pi}{2\sqrt{2}} \ln\left(\sqrt{2} + 1\right).$$

*(c)* Prove that

$$\int_0^1 \sqrt{1 - x^4}\,dx = \frac{1}{6\sqrt{2\pi}}\left(\Gamma\left(\frac{1}{4}\right)\right)^2$$

(3+3+3) **(CO4)**

**8.** *(a)* Prove that

$$\int_a^b (x-a)^{p-1}(b-x)^{q-1}\,dx = (b-a)^{p+q-1}\beta(p,q),$$

$p > 0, q > 0.$