

(6)

4. Find the general solution of the following system :

$$\frac{dx}{dt} = x + 2y$$

$$\frac{dy}{dt} = 3x + 2y$$

[CO4] 6

★ ★ ★

Ex/SC/MATH/UG/MINOR/TH/21/202A/2025

BACHELOR OF SCIENCE EXAMINATION, 2025

(2nd Year, 1st Semester)

MATHEMATICS

PAPER : MINOR/TH/21/202A

(Mathematics II)

Time : Two Hours

Full Marks : 40

Use separate Answer-Scripts for each Part.

Symbols / Notations have their usual meanings.

PART—I (12 marks)

Answer *any two* questions from **1** to **3** and **one** question from **4** to **5**.

1. If $x = \cos \alpha + i \sin \alpha$, $y = \cos \beta + i \sin \beta$, $z = \cos \gamma + i \sin \gamma$ and $x + y + z = xyz$, then show that $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) = -1$. [CO2] 4

2. Show that the product of all values of $(\sqrt{3} + i)^{3/5}$ is $8i$. [CO2] 4

(2)

3. (a) Apply Descartes's rule of signs to find the nature of the roots of the equation $x^9 + x^3 = 0$.

- (b) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$ ($r \neq 0$), then find the equation

whose roots are $\alpha - \frac{\beta\gamma}{\alpha}, \beta - \frac{\gamma\alpha}{\beta}, \gamma - \frac{\alpha\beta}{\gamma}$.

2+2 [CO2]

4. Solve by Cardan's method $x^3 - 27x - 54 = 0$. 4 [CO1]

5. Solve the equation $16x^4 - 64x^3 + 56x^2 + 16x - 15 = 0$, whose roots are in arithmetic progression. 4 [CO1]

PART—II (12 marks)

Attempt Question No. 4 and *any two* from the rest : 4×3=12

CO1

1. (a) Find all subgroups of the permutation group S_3 . Is union of any two subgroups of S_3 a subgroup? Justify with an example.

- (b) Find the order of the permutation (1 2 3 4) (5 6).

(1+2+1)

MATH-1052

[Continued]

(5)

(OR)

- (b) Find for what values of λ and a , the following system of equations

$$x + 4y + 2z = 1$$

$$2x + 7y + 5z = 2\lambda,$$

$$4x + ay + 10z = 2\lambda + 1,$$

has (i) a unique solution (ii) no solution. [CO3] 4

2. Solve the differential equation $\frac{d^2y}{dx^2} - y = xe^x \sin x$.

[CO4] 6

3. Solve the following differential equations (*any two*) :

(a) $x \frac{dy}{dx} + y = y^2 \log x$

(b) $y = px - p^2$

(c) $(x^4 y^2 - y)dx + (x^2 y^4 - x)dy = 0$ [CO4] 3+3

MATH-1052

[Turn Over]

(3)

CO1

2. Consider the rings $(\mathbb{Z}, +, \cdot)$ and $(\mathbb{Z}_6, +_6, \cdot_6)$. Let $f: \mathbb{Z} \rightarrow \mathbb{Z}_6$ be defined by $f(n) = [n]$, for all $n \in \mathbb{Z}$. Show that f is a ring homomorphism and find also Kernel of f (i.e., $\ker f$), where $[n]$ denotes the residue classes of integers modulo n . (2+2)

CO1

3. Let S and T be the two ideals of a ring R , then prove that $S \cap T$ is also an ideal of R .

Illustrate with an example that union $S \cup T$ of two ideals, need not be an ideal of R . (2+2)

CO2

4. (a) Suppose S be the subset of $\mathbb{R}^3$, defined by $S = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 1\}$. Is S subspace of $\mathbb{R}^3$? Explain.
- (b) Find the dimension of the subspace $S \cap T$ of $\mathbb{R}^4$, where $S = \{(x, y, z, w) \in \mathbb{R}^4 : x + y + z + w = 0\}$ and $T = \{(x, y, z, w) \in \mathbb{R}^4 : 2x + y - z + w = 0\}$. (1+3)

MATH-1052

[Turn Over]

(4)

(OR)

CO2

4. (a) Is the union of two subspaces of a vector space V over a field F , a subspace? Explain with an example.
- (b) Find the dimension of the subspace $P \cap Q$, where P and Q are subspaces of the vector space $\mathbb{R}^4$ given by $P = \{(x, y, z, w) \in \mathbb{R}^4 : 2x + y + 3z + w = 0\}$ and $Q = \{(x, y, z, w) \in \mathbb{R}^4 : x + 2y + z + 3w = 0\}$. (1+3)

PART—III (16 marks)

Answer Question No. 1 and any two questions from the following :

16

1. (a) Reduce the matrix A to row reduced echelon form, where

$$A = \begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$

and hence find its rank.

[CO3] 4

MATH-1052

[Continued]