(b) Find the radial and cross-radial components of velocity and acceleration of a particle moving in a plane in polar co-ordinates (r, θ).

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BACHELOR OF SCIENCE EXAMINATION, 2025

(2nd Year, 1st Semester)

Mathematics

(ELEMENTARY MECHANICS)

Time : *Two Hours* *Full Marks* : 40

The figures in the margin indicate full marks.

(Symbols / Notations have their usual meanings)

1. [CO1] (Answer *any* ***one***) : 1×8=8

(a) If a system of forces in one plane reduces to a couple of moment G and when each force is turned round its point of application through a right angle, the system reduces to a couple to moment H. Prove that when each force is turned through an angle 30º the system is equivalent to a couple whose moment is $\frac{1}{2}\left(\sqrt{3}G + H\right)$.

(b) If a particle of mass m be acted on by equal constant forces mf in the directions of the tangent and normal to its path and if the resistance be mk^2v^2f, prove that the intrinsic equation of the path is

$$e^{2k^2 fs} - 1 = k^2u^2(e^{2\psi} - 1).$$

2. [CO2] (Answer *any* ***one***) : 8×1=8

(a) Two forces P and Q act along the straight lines whose equations are $y = x\tan\alpha,\ z = c$ and $y = -x\tan\alpha,\ z = -c$ respectively. Show that their central axis is

$$y = x\frac{P-Q}{P+Q}\tan\alpha,\quad \frac{z}{c} = \frac{P^2 - Q^2}{P^2 + 2PQ\cos 2\alpha + Q^2}$$

for all values of P and Q, also prove that this line is a generator of the surface $(x^2 + y^2)z\sin 2\alpha = 2cxy$.

(b) A rod AB is movable about a point A and B is attached to a string whose other end is tied to a ring. The ring slides along a smooth horizontal wire passing through A. Prove, by the principle of virtual work that the horizontal force necessary to keep the ring at rest is

$$\frac{W\cos\alpha\cos\beta}{2\sin(\alpha+\beta)},$$

where W is the weight of the rod and α, β the inclinations of the rod and the string to the horizontal.

3. [CO3] (Answer *any* ***two***) : 8×2=16

(a) A particle moves in a straight line under a force to a point is it varying as (distance)$^{-\frac{4}{3}}$. Show that the velocity in falling from rest at infinity to a distance a is equal to that acquired in falling from rest at a distance a to a distance $a/8$.

(b) A particle describes an ellipse under an acceleration which is always directed perpendicularly towards the major axis. Find the law of force.

(c) A uniform rod of length $2l$ is attached by a smooth ring at the ends to a parabolic wire, fixed with its axis vertical and vertex downwards and of latus rectum 4a. Show that the angle θ which the rod makes with the horizon in a slanting position of equilibrium is given by $\cos^2\theta = \frac{2a}{l}$, and that, if these positions exist, they are stable.

4. [CO4] (Answer *any* ***one***) : 8×1=8

(a) Show that the distance from the cusp of the centroid of the area of the cardioid $r = a(1+\cos\theta)$, when the density at any point varies as the square of the distance from the cusp is $\frac{6}{5}a$.