B. Sc. Mathematics Examination, 2025 (OLD)

(2nd Year, 1st Semester)

(Mathematics–II)

Paper : GE-03

Time : *Two Hours* *Full Marks* : 40

Use a separate answer scripts for each part.

Symbols / Notations have their usual meanings.

PART—I (12 Marks)

Answer *any* **three** questions : 4×3=12

1. Find mod z and principal amplitude of

$$z = 1 - \sin\theta(\sin\theta + i\cos\theta)\ \frac{\pi}{2} < \theta < \pi$$

2. Find the rank of A, where

$$A = \begin{pmatrix} 8 & 1 & 3 & 6 \\ -8 & -1 & -3 & 4 \\ 0 & 3 & 2 & 2 \end{pmatrix}$$

3. Show that the roots of the following equation are all real

$$\frac{1}{x+a_1} + \frac{1}{x+a_2} + \cdots + \frac{1}{x+a_n} = \frac{1}{x+6}$$

where $a_i, b \in \mathbb{R}^+$ and $b < a_i\ \forall i$.

4. Solve the equation $x^4 - 2x^2 + 8x - 3 = 0$ by Ferrari's method.

5. If $\cos\alpha + \cos\beta + \cos\gamma = 0$ and $\sin\alpha + \sin\beta + \sin\gamma = 0$, then using the property of complex number show that

$$\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$$

PART—II (16 Marks)

Answer *any* **four** questions : 4×4=16

1. Solve the equation

$$(x^2 + 2xy - y^2)dx + (y^2 + 2xy - x^2)dy = 0,$$

given $y = 1$ when $x = 0$.

2. Solve $y(1 + xy)dx + x(1 - xy)dy = 0$.

3. Solve the differential equation $\cos^2 x \dfrac{dy}{dx} + y = \tan x$.

4. Find the general solution and the singular solution of the ODE :

$$(y - px)(p - 1) = p, \text{ where } p = \frac{dy}{dx}$$

5. Solve : $(2D^2 - 5DD' + 2D'^2)z = 5\sin(2x + y)$.

6. Check the exactness of the ODE :

$$(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0,$$

and then solve it.

PART—III (12 Marks)

Answer *any* **three** questions : 4×3=12

1. Find the real root of the equation $3x - \cos x - 1 = 0$ in the interval $[0, \pi/2]$ by iteration method correct to three significant figures. 4

2. Use the Newton-Raphson method to find the positive real root of the equation $x^3 - x - 1 = 0$ correct up to three decimal places. 4

3. Solve the following system of equations

$$5x + 2y + z = 12;\ x + 4y + 2z = 15;\ x + 2y + 5z = 20$$

by Gauss-Seidel method where the initial approximations of variables are given $x_0 = 0,\ y_0 = 0,\ z_0 = 0$. 4

4. Calculate the approximate value of $\int_{-3}^{3} x^4 dx$ using

   (i) Trapezoidal rule and

   (ii) Simpson's one-third rule, by taking six equal sub-intervals. 2+2=4

5. Solve by Picard's method to obtain the solution of the differential equation $\dfrac{dy}{dx} = x + y^2$ for $x = 0{\cdot}4$, where $y(0) = 0$, correct up to four decimal places. 4

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