Bachelor of Science Examination, 2025

(Third Year, First Semester)

MATHEMATICS

Paper : CORE 11

(Partial Differential Equation)

Time : 2 *Hours* *Full Marks* : 40

Use separate Answer-Scripts for each Part.

The figures in the margin indicate full marks

Symbols/Notations have their usual meanings.

PART—I

(20 Marks)

Answer Question No. **1** and *any* **three** from the rest

1. **(CO 1)** Explain the geometrical significance of the first order partial differential equation $Pp + Qq = R$. 5

2. **(CO 1)** Show that all characteristic curves of the partial differential equation

$$(2x+z)\,p + (2y+z)\,q = z$$

through point (1,1) are same straight line. Find the equation of the straight line. 5

3. **(CO 4)** Find the integral surface of $x^2p + y^2q + z^2 = 0$ which passes through the hyperbola $xy = x + y,\ z = 1$. 5

4. **(CO 3)** Find the general solution of

$$(x^2 - y^2 - z^2)\,p + 2xyq = 2xz\,.$$ 5

5. **(CO 3)** Find the complete solution and singular solution (if any) of $p(1-q^2) = q(1-z)$. 5

6. *(a)* **(CO 3)** Use the method of separation of variables to solve $p^2 + q^2 = 1$. 3

(b) **(CO 2)** Solve $py + q = x$ by reducing to canonical form. 2

PART—II
(20 Marks)

Answer *any* **two** questions

1. *(a)* **(CO1)** Suppose a string of length l is stretched under tension and fastened at two end points. The string is flexible and it is set in motion at time $t = 0$. If $u(x, t)$ denotes the vertical displacement of the string at position x, at time t, derive the differential equation satisfied by $u(x, t)$. 4

(b) **(CO2)** Reduce canonical form of the equation

$$\frac{\partial^2 u}{\partial x^2} + y^2\frac{\partial^2 u}{\partial y^2} = 0 \text{ for } y \neq 0.$$ 6

2. **(CO3)** Solve

$$u_{tt} - c^2u_{xx} = 2 \qquad -\infty < x < \infty,\ t > 0$$

$$u(x,0) = x^2,\ u_t(x,0) = \cos x, \qquad -\infty < x < \infty,$$

c^2 being a constant. 10

3. **(CO4)** Use separation of variables method to find solution of the problem

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < 10,\ t > 0$$

$$u(0,t) = u(10,t) = 0, \qquad t > 0$$

$$u(x,0) = \sin\left(\frac{\pi x}{10}\right), \qquad 0 < x < 10,$$

where $k > 0$ is a constant. 10

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