Bachelor of Science Examination, 2025

(3rd Year, 1st Semester)

MATHEMATICS (HONOURS)

Paper : DSE 1A

(Linear Programming and Game Theory)

Time : *Two Hours* | *Full Marks* : 40

Use separate Answer-Scripts for each Part.

The figures in the margin indicate full marks

(Symbols/Notations have their usual meanings)

PART—I (20 Marks)

Answer *any* **two** questions :

1. *(a)* Find all basic feasible solutions of the following system :

$$2x_1 + 3x_2 - x_3 + 4x_4 = 8$$
$$-x_1 + 2x_2 - 6x_3 + 7x_4 = 3$$
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(b) Solve the linear programming by two-phase method :

Maximize $Z = 4x_1 + 3x_2$

Subject to $x_1 + 2x_2 \geq 8,$

$3x_1 + 2x_2 \geq 12,$

$x_1, x_2 \geq 0.$ 5

2. (a) Reduce the following linear programming problem to standard form :

Minimize $Z = 2x_1 - 3x_2 + x_3$

Subject to $|x_1 + 2x_2 - x_3| \le 4$,

$3x_1 - x_2 + 2x_3 \ge 5$,

$5x_1 + 3x_2 - x_3 = 8$.

$x_1, x_2 \ge 0$, x_3 is an unrestricted in sign. 4

(b) Solve the linear programming by Charnes Big-M method :

Maximize $Z = 5x_1 - 2x_2 + 3x_3$

Subject to $2x_1 + 2x_2 - x_3 \ge 2$,

$3x_1 - 4x_2 \le 3$,

$x_2 + 3x_3 \le 5$,

$x_1, x_2, x_3 \ge 0$. 6

3. (a) Find the dual of the following linear programming problem :

Minimize $Z = 2x_1 + 3x_2 - x_3$

Subject to $2x_1 - x_2 + x_3 = 4$,

$|x_1 + x_2 - x_3| \le 3$,

$2x_2 - x_3 \ge -1$.

$x_1, x_2 \ge 0$, x_3 is an unrestricted in sign. 4

(b) Two companies *A* and *B* are competing for the same product. Their different strategies are given in the following pay-off matrix :

Player *B*

Player *A*		B_1	B_2
	A_1	2	–3
	A_2	–2	5
	A_3	3	–1

Use linear programming technique to determine the best strategies for both the players. 5+5

4. (a) *A* and *B* play a game in which each has three coins, a 5p, a 10p and a 20p. Each selects a coin without the knowledge of the other's choice. If the sum of the coins be an odd amount, then *A* wins *B*'s coin; if the sum be even, then *B* wins *A*'s coin. Find the best strategy for each player and the value of the game.

(b) Solve the following game problem graphically :

Player *B*

Player *A*		B_1	B_2	B_3	B_4
	A_1	1	3	0	2
	A_2	3	0	1	–1

5+5

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(b) Use duality to solve the following linear programming problem :

$$\begin{aligned} \text{Minimize} \quad & Z = x_1 + 2x_2 + 3x_3 \\ \text{Subject to} \quad & x_1 - x_2 + x_3 \geq 4, \\ & x_1 + x_2 + 2x_3 \leq 8, \\ & x_2 - x_3 \geq 2, \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

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PART—II (20 Marks)

Answer *any* **two** questions :

1. (a) Define transportation problem. Show that the number of basic variables in a transportation problem is at most $(m+n-1)$, m and n being the number of origins and destinations.

(b) A team of 4 horses and 4 riders has entered in a jumping show contest. The number of penalty points to be expected when each rider rides any horse is shown below :

Riders

Horses	R_1	R_2	R_3	R_4
H_1	2	3	7	6
H_2	4	1	5	2
H_3	6	8	1	2
H_4	4	2	5	7

How should the horses be allotted to the riders so as to minimize the expected loss of the team? 5+5

2. (a) Is assignment problem a type of transportation problem? Justify your answer.

Show that if a cost be added to any row of the cost matrix of an assignment problem, then the resulting assignment problem has the same optimal solution as the original problem.

(b) Is

	1	2	3	4
1			50	20
2	55			
3	30	35		25

an optimal solution of the following transportation problem? If not, modify it to obtain a better feasible solution.

	1	2	3	4	a_i
1	6	1	9	3	70
2	11	5	2	8	55
3	10	12	4	7	90
b_j	85	35	50	45	

5+5

3. (a) Solve the following travelling salesman problem :

To

From	A	B	C	D
A	∞	15	30	4
B	6	∞	4	1
C	10	15	∞	16
D	7	18	13	∞