**5.** Find the approximate confidence interval of $p$ for a binomial $(n, p)$ distribution by considering a single observation. **[CO2]**

**6.** Find the maximum likelihood estimate of $(m, \sigma)$ of normal $(m, \sigma)$ population on the basis of a sample of size 100 : 1, 2, ....., 100. **[CO3]**

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**Ex/SC/MATH/UG/DSE/TH/02/A/2025**

## BACHELOR OF SCIENCE EXAMINATION, 2025

**( 3rd Year, 1st Semester )**

## MATHEMATICS

**PAPER : DSE/TH/02/A**

**( Probability and Statistics )**

*Time* : *Two Hours* *Full Marks* : 40

*Use separate Answer-Scripts for each Part.*

## PART—I

**( 20 Marks )**

Attempt *any* **two** questions, Each question carries 10 marks : $10\times2=20$

**1.** Consider a fair Dice with misprints so that its six faces are marked as 1, 2, 3, 4, 5, 5. Let this dice be rolled repeatedly and independently. Find the probabilities of the following events :

*(a)* In the $10^{th}$ roll, the face number 4 comes up for the first time.

*(b)* In the $12^{th}$ roll, the number 4 or the number 5 comes up for the first time.

*(c)* In the $18^{th}$ roll, both 4 and 5 come up for the first time [This means, upto the $17^{th}$ roll, both the numbers 4 and 5 did not come up]. $4+2+4 = 10$ **[CO1]**

**2.** *(a)* Define a $\chi^2$ (Chi-Square) random variable with $n$ degrees of freedom (d.f.).

*(b)* Find the probability density function of a $\chi^2$ random variable with 2 d.f. and find its expectation.

4+6 = 10 **[CO2]**

**3.** *(a)* Find the mean and variance of a Poisson random variable with parameter $\sqrt{2}$. **[CO3]**

*(b)* In an experiment, it is observed that the moment generating function of a random variable is $M(t) = \left(\frac{1}{4}e^t + \frac{3}{4}\right)^{100}$. Find the mean and variance of the random variable. Can you identify the random variable? **[CO4]** 5+5 = 10

## PART—II

### ( 20 Marks )

*Symbols/Notations have their usual meanings.*

Answer *any* **four** questions : 5×4=20

**1.** Define the following items with respect to a parameter of a distribution :

*(a)* Good estimate,

*(b)* Consistent estimate,





*(c)* Unbiased estimate,

*(d)* Positively biased estimate,

*(e)* Negatively biased estimate. **[CO1]**

**2.** Show that the sample mean is a consistent and unbiased estimate of the population mean by considering a sample of size $n$. **[CO2]**

**3.** If $W = \{x : x \geq 1\}$ is the critical region for testing the hypothesis $H_0 : \theta = 2$ against the alternative hypothesis $H_1 : \theta = 1$ on the basis of the single observation $x$ from the population having probability density function

$$f(x) = \theta e^{-\theta x},\ 0 \leq x < \infty$$

for the unknown parameter $\theta$, then find the probability of Type-I error and the power of the test. **[CO4]**

**4.** Let $x_1, x_2, \ldots\ldots, x_n$ be a sample of size $n$ drawn from the population of a random variable $X$. If $Var(X)$ exists finitely, then find the value of $d$ such that the statistic $\sum_{i=1}^{n} d\left(x_i - \bar{x}\right)^2$ is an unbiased estimate of the population variance, where $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$. **[CO3]**