Master of Science Examination, 2024

(1st Year, 1st Semester)

Mathematics

Paper : CORE – 05

(Differential Geometry, Probability and Stochastic Process)

Time : 2 *Hours* *Full Marks* : 40

Use separate Answer-Script for each Part.

PART—I (20 Marks)

Answer **any *four*** of the following : 20

1. *(a)* Find the cylindrical polar coordinates of the point whose rectangular Cartesian coordinate is $(1, \sqrt{3}, 1)$.

(b) A covariant vector has components $\left(2x, y^2 - z, z^2\right)$ in rectangular coordinates. Determine its covariant components in cylindrical coordinates. 2+3=5

2. *(a)* Show that Kronecker delta is a mixed tensor of type (1, 1).

(b) If A_m is a covariant vector, show that $\frac{\partial A_m}{\partial x^s}$ is not a tensor but $\frac{\partial A_m}{\partial x^s} - \frac{\partial A_s}{\partial x^m}$ is a skew symmetric covariant tensor of type (0, 2). 1+4=5

3. (a) Prove that $\left\{ \begin{matrix} & i & \\ i & & j \end{matrix} \right\} = \frac{\partial}{\partial x^j}\left(\log\sqrt{g}\right),\ g = |g_{ij}| \neq 0$.

(b) Find the Riemann curvature tensor for the space whose metric is $ds^2 = \left(dx^1\right)^2 + \left(dx^2\right)^2 + \left(dx^3\right)^2 - c^2\left(dx^4\right)^2$. 3+2=5

4. Establish Serret–Frenet formulae for space curve in tensor approach. 5

5. (a) Define Bertrand curves and state a relation between its curvature and torsion.

(b) On a surface whose parametric equation is given by $x^1 = a\sin u\cos v,\ x^2 = a\sin u\sin v,\ x^3 = a\cos u$, find the first fundamental form. Also show that the parametric curves are orthogonal. 2+3=5

6. (a) When a surface is said to be developable? Determine whether the surface with the metric $ds^2 = v^2du^2 + u^2dv^2$ is developable or not.

(b) Write Gauss's formula for a surface. 4+1=5

PART—II (20 Marks)

Answer ***any two*** questions : 20

1. (a) Define *(i)* a normal number and *(ii)* a negligible set as per E. Borel.

(b) Prove that the set of normal numbers is uncountable. 5+5=10

2. (a) State Borel's Normal Number Theorem.

(b) State and prove Borel's Weak Law of Large Numbers using the arguments of Borel. 3+7=10

3. (a) Define *(i)* a Stochastic process, *(ii)* a chain and *(iii)* a Markov chain.

(b) Let $P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{9}{10} & \frac{1}{10} & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & 0 & 1 \end{bmatrix}$ be the One-Step transition matrix of a Markov chain. Find all the recurrent and transient states of this chain. 5+5=10

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