Master of Science Examination, 2024

(1st Year, 1st Semester)

Mathematics

Paper : Core - 04

(General Mechanics)

Time : 2 *Hours* *Full Marks* : 40

The figures in the margin indicate full marks.

Symbols have their usual meanings.

Answer ***any four*** *questions.* 10×4=40

1. *(a)* Show that the K.E. of a moving system can be expressed as

$$T = \frac{1}{2}\sum_{i,j} a_{ij}\, \dot{q}^i\, \dot{q}^j + \sum_i b_i\, \dot{q}^i + c$$

(b) The K.E. and P.E. of a particle are given by $T = \frac{1}{2}(\dot{x}^2 + \dot{y}^2)$ and $V = \frac{\alpha}{x^2} + \frac{\beta}{y^2} + \frac{\gamma}{r} + \frac{\gamma'}{r'} + \delta(x^2 + y^2)$, where $\alpha, \beta, \gamma, \gamma'$ and δ are constants and r, r' are the distances of the particle from the points $(\mu, 0)$ and $(-\mu, 0)$ respectively, μ being a constant. Show that the system can be converted into Liouville's type. 4+6=10

2. (a) Define action of a mechanical system. State and prove the principle of least action.

(b) What do you mean by Legendre's dual transformation?

(c) Derive Hamilton-Jacobi partial differential equation. (1+3)+2+4=10

3. (a) Define canonical transformation. Show that Poisson bracket remains invariant under canonical transformation.

(b) Show that the transformation

$$Q = \sqrt{q}\cos p, P = \sqrt{q}\sin p$$

represents a canonical transformation. Hence evaluate the new Hamiltonian of the system for which

$$T = \frac{1}{2}m\dot{q}^2, V = \frac{1}{2}kq^2$$ (1+4)+(3+2)=10

4. (a) Show that $J_1 = \int_{S_2} \sum dq_i \, dq_p$ is invariant under canonical transformation where S_2 is a 2D surface in phase space.

(b) Establish Euler's dynamical equations from the principle of angular momentum. 5+5=10

5. (a) Establish Earth's rotation using Foucault's pendulum.

(b) In a dynamical system with two d.f.; the K.E. and P.E. are given by

$$T_1 = \frac{\dot{q}_1^2}{2(a + bq_2)} + \frac{1}{2}q_2^2\dot{q}_2^2, \; V = c + dq_2$$

where a, b, c and d are constants. Show that the value of q_2 in terms of time is given by the equation

$$(q_2 - k)(q_2 + 2k)^2 = h(t - t_0)$$

with h, k and t_0 as constants. 5+5=10

6. (a) Establish Hamilton's principle from D'Alembert's principle.

(b) Show that the time periods of small oscillation of a constraint physical system always lie between the corresponding time periods of the unconstrained system. 5+5=10

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