Master of Science Examination, 2024

(Second Year, First Semester)

Mathematics

Paper : DSE - 04 A25

(Modelling of Biological Events)

Full Marks : 40 *Time* : 2 *Hours*

Symbols have their usual meanings.

Use separate answer script for each part.

Answer Q. No. **4** and ***any two*** from the rest.

PART—I

1. Distinguish between DDE and ODE. Suppose the instantaneous death of a species having density N is proportional to its density at T time unit before. Write a rate equation for the death of this species. Considering $T = 1$, find the species density for $t \in [2, 3]$ when the history function is given as $N(t) = 10$ for $t \in [-1, 0]$. [2+1+7]

2. The Holling-Tanner type predator-prey model with diffusion is represented by

$$\frac{\partial N}{\partial t} = rN\left(1 - \frac{N}{k}\right) - \frac{eNP}{m+N} + D_1 \frac{\partial^2 N}{\partial x^2},$$

$$\frac{\partial P}{\partial t} = \theta P\left(1 - \frac{gP}{N}\right) + D_2 \frac{\partial^2 P}{\partial x^2},$$

where $N(x, t)$ and $P(x, t)$ are the prey and predator densities at position x at time t. All parameters are positive and carry the usual meanings. Assuming positive initial values and zero-flux boundary conditions, discuss the stability and diffusion-driven instability (if any) of the homogeneous coexisting equilibrium point. [10]

3. Consider the chemical reactions

$X + Y \xrightarrow{\kappa_1} P,\ P \xrightarrow{\kappa_2} X + Y,\ \kappa_1, \kappa_2$ are the rate constants, with the initial concentrations

$$[X(0)] + [P(0)] = 2, [Y(0)] - X(0)] = 1.$$

Using the mass action law, construct the rate equations of the reactants. Assuming $\kappa_1 = 1$ and $\kappa_2 = 2$, find the behaviour of the concentrations of the reactants when $t \to \infty$. [2+8]

4. Construct an age-structured predator-prey model, where predator species have two age classes. [4]

PART—II

(16 Marks)

Answer ***any two*** questions.

1. Write down a discrete mathematical model of modified Lotka-Volterra model. Determine the steady states and discuss the stability properties of the interior equilibrium. 8

2. Write down a discrete logistic growth prey-predator model with Holling type-II functional response. Determine the steady states and discuss the stability properties of the interior equilibrium. 8

3. Describe Yule-Furry model (pure birth process) of stochastic epidemic model. Also, find the expected number of infectives in the population at any time t. 8

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