Ex/SC/MATH/PG/DSE/TH/03/A8/2024

Master of Science Examination, 2024

(2nd Year, 1st Semester)

Mathematics

Paper : DSE - 03 (A8)

(Solid Mechanics - 1)

Full Marks : 40 *Time* : *Two Hours*

The figures in the margin indicate the full marks.

(*Symbols / Notations have their usual meanings.*)

Answer ***any four*** questions

1. Show that stress strain relation for isotropic elastic medium can be written as

$$\tau_{ij} = \lambda\delta_{ij}e_{ii} + 2\mu e_{ij}$$

Also show that for infinitesimal deformation of an isotropic elastic body the strain energy density function U can be expressed as

$$2U = (\lambda + 2\mu)I_1^2 - 4\mu I_2$$

where $I_i (i = 1, 2)$ are strain invariants. 10

2. Deduce compatibility equations in the form

$$e_{ij,kl} + e_{kl,ij} - e_{ik,jl} - e_{jl,ik} = 0$$

Hence deduce the Beltrami-Michell compatibility equations in the form

$$\nabla^2\tau_{ij} + \frac{1}{1+\sigma}\Theta_{,ij} = -\frac{\sigma}{1+\sigma}\delta_{ij}\, div\, \overline{F} - (F_{i,j} + F_{j,i})$$

where $\Theta = \tau_{ii}$ and F_i is the components of body force.

If field of force $\vec{F} = \vec{\nabla}\varphi$, then show that the above equation can be written as

$$\nabla^2\tau_{ij} + \frac{1}{1+\sigma}\Theta_{,ij} = -\frac{\sigma}{1+\sigma}\delta_{ij}\nabla^2 - 2\varphi_{,ij}$$ 10

3. *(a)* Show that principal directions of strain at each point of a linearly elastic isotropic body are coincide with the principal directions of stress. 5

(b) State and prove Clapeyron's theorem. 5

4. Prove that the problem of torsion of a beam by the terminal couples can be reduced either to a Neumann problem or to a Dirichlet problem. Hence find the resultant force and resultant moment of the external forces applied to the end of the beam. Also show that the torsional rigidity depends on the modulus of rigidity and shape of the cross-section. 10

5. A cylinder of uniform thickness is rotating about an axis through the centre with angular velocity ω. Show that the stresses are greatest at the centre of the cylinder. Show also that by making a small tubular cylinder at the centre of the cylinder, the maximum tangential stress approaches a value twice as great as that for a solid cylinder (assuming that there is no external forces applied at the boundaries). 10

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