

Ex/SC/MATH/PG/DSE/TH/04/A27/2024

MASTER OF SCIENCE EXAMINATION, 2024

(2nd Year, 1st Semester)

MATHEMATICS

PAPER : DSE 04-A – 27

( Solid Mechanics – II )

Time : 2 Hours

Full Marks : 40

*Symbols/Notations have their usual meanings.*

Answer **any four** questions :

1. Apply Helmholtz's theorem to decompose the displacement equation of motion, in absence of body forces, of an isotropic elastic body in the form : 10

$$\nabla^2 \phi = \frac{1}{c_1^2} \frac{\partial^2 \phi}{\partial t^2},$$
$$\nabla^2 \vec{\psi} = \frac{1}{c_2^2} \frac{\partial^2 \vec{\psi}}{\partial t^2}.$$

2. Show that the plane wave solution of the form :

$$\vec{U} = \vec{d} F(\vec{r} \cdot \vec{N} - ct)$$

to the equation of motion

$$\rho \frac{\partial^2 \vec{U}}{\partial t^2} = (\lambda + \mu) \vec{\nabla} (\vec{\nabla} \cdot \vec{U}) + \mu \nabla^2 \vec{U}$$

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is possible if (i) either  $\vec{d} = \pm \vec{N}$  or (ii)  $\vec{d} \cdot \vec{N} = 0$  and  $\mu = \rho c^2$ .

Also explain the cases.

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3. A uniform pressure varying harmonically with time is acting radially on the surface of a cylindrical cavity of radius 'a' in an infinite elastic medium. Determine the displacement at any point of the medium. 10
4. Deduce the equation of flexural vibration of a thin rod, when the rotatory inertia effect is taken into account. 10
5. Show that the face velocity 'c' of Rayleigh wave satisfy the following equation : 10

$$\left(2 - \frac{c^2}{c_2^2}\right)^2 - 4 \left(1 - \frac{c^2}{c_1^2}\right)^{1/2} \left(1 - \frac{c^2}{c_2^2}\right)^{1/2} = 0,$$

where  $c_1$  and  $c_2$  are longitudinal and shear wave velocities respectively.

6. Show that Love waves are dispersive waves. Find also the conditions in terms of velocities for which Love waves exist. 10

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