Master of Science Examination, 2024

(2nd Year, 1st Semester)

Mathematics

Paper : DSE - 03 A5

[Fluid Mechanics - I]

Time : *Two Hours* *Full Marks* : 40

The figures in the margin indicate the full marks.

Notations / Symbols have their usual meanings.

Answer question no. ***1*** *and* ***any two*** *from the rest.*

1. *(a)* Derive the equation of continuity, in its most general form, using Eulerian co-ordinate system. Hence find the corresponding equation for a liquid. Give the physical interpretation of the equation of continuity. 8

Or,

(b) Prove that if,

$$\lambda = \frac{\partial u}{\partial t} - v\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + w\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

and μ, γ have two similar expressions, then $\lambda\, dx + \mu\, dy + \gamma\, dz$ is a perfect differential, if the forces are conservative and the density is constant. 8

2. *(a)* For a two-dimensional irrotational motion of a liquid, show that φ, ψ are harmonic functions, and the family of curves φ = constant and ψ = constant intersect orthogonally. 4

(b) Show that the stream function ψ for the most general motion of a cylinder moving in an infinite mass of liquid is given by

$$\psi = -Uy + Vx + \frac{\omega}{2}(x^2 + y^2) + c. \qquad 5$$

(c) Show that when a cylinder moves uniformly in a given straight line in an infinite liquid, the path of any point in the liquid is given by the equations

$$\frac{dz}{dt} = \frac{va^2}{(\bar{z} - vt)^2}, \quad \frac{d\bar{z}}{dt} = \frac{va^2}{(z - vt)^2},$$

where v is the velocity of the cylinder, a its radius, $z, \bar{z}$ are $x + iy$, $x - iy$, where x, y are the coordinates measured from the starting point of the axis, along and perpendicular to its direction of motion. 7

3. *(a)* State and prove the theorem of permanence of irrotational motion. 7

(b) Within a circular boundary of radius 'a' there is a two-dimensional liquid motion due to a source producing liquid at a rate m, at a distance f from the centre and an equal sink at the centre. Find the complex potential and show that the resultant of the pressure on the boundary is

$\dfrac{\rho m^2 f^3}{2a^2\pi(a^2 - f^2)}$, where ρ is the density. 9

4. *(a)* Show that the group velocity of—

(i) deep sea water is half of the wave velocity;

(ii) shallow water is equal to the wave velocity. 8

(b) Consider an infinite row of parallel vortices given by $x = na$, $y = b$ of strength κ, and $x = na$, $y = -b$ of strength $-\kappa$, where n is any positive or negative integer or zero. Show that if the liquid at infinity is at rest, the vortices moves as a whole, in the direction of its length with the speed $\dfrac{\kappa}{2a}\coth\left(\dfrac{2\pi b}{a}\right)$. 8

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