MASTER OF SCIENCE EXAMINATION, 2024

(Second Year, First Semester)

MATHEMATICS

PAPER : DSE - 02

[Numerical Analysis - I (Theory)]

Time : *One hour fifteen minutes* *Full Marks* : 24

Symbols / Notations have their usual meanings.

Use separate answer script for each Group.

GROUP—A

(Marks : 16)

Answer ***any two*** questions.

1. *(a)* Define cubic spline interpolation with all its properties in the case of natural boundary condition.

(b) Write an algorithm to construct natural cubic spline from a given set of points (x_i, y_i) with $i = 0, 1, \cdots, n$. 4+4

2. *(a)* Construct an objective function for the linear conjugate gradient method to solve a linear system $AX = b$, and hence show that the gradient of the objective function is equal to the residual of the linear system.

(b) Consider a linear system $AX = b$, where A is an $n \times n$ matrix. Show that if A is strictly diagonally dominant, then $\| H_J \|_\infty < 1$, where H_J is the iteration matrix for Jacobi method. 4+4

3. *(a)* Determine the operational count of Gauss-Jordan method for finding inverse of a square matrix of order n.

(b) Prove that

$$\frac{1}{\kappa(A)}\frac{\|r\|}{\|b\|} \le \frac{\|X-\tilde{X}\|}{\|X\|} \le \kappa(A)\frac{\|r\|}{\|b\|},$$

where $\kappa(A)$ is the condition number of a linear system $AX = b$ and r is the residual for an approximate solution $\tilde{X}$. How do you define a linear system is ill-conditioned? 4+4

GROUP—B

(Marks : 8)

Answer ***any one*** question.

4. Find all eigenvalues and corresponding eigenvectors of the matrix

$$\begin{pmatrix} 1 & \sqrt{2} & 2 \\ \sqrt{3} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{pmatrix}$$

by Jacobi method. 8

5. *(a)* Estimate the eigenvalues of the following matrix using Gerschgorin bounds :

$$\begin{pmatrix} 1 & 2 & -1 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$$

(b) Prove that the largest eigenvalue in modulus of a square matrix cannot exceed the largest sum of the moduli of the elements along any row or column. 3+5

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