M. Sc. Mathematics Examination, 2024

(2nd Year, 2nd Semester, Supplementary)

Mathematics

Paper – DSE-05A

[Numerical Analysis - II (Theory)]

Time : 1 hour 15 minutes Full Marks : 24

(Use separate answer script for each Part)

Part – I (Marks: 16)

Answer ***any two*** questions.

Symbols and notations have their usual meanings.

1. Solve the following boundary value problem using a second order method

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2, \qquad 0 < x < 1$$
$$y'(0) - y(0) = 1, \;\; y'(1) + y(1) = 1$$

with step length $h = 0{\cdot}25$. 8

2. Solve the following two-dimensional elliptic mixed BVP

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0; \;\; 0 \le x \le 1, \;\; 0 \le y \le 1$$

subject to the boundary conditions

$$u(x,0) = 2x, \;\; u(x,1) = 2x - 1; \;\; 0 \le y \le 1$$

$$u(0,y) + \frac{\partial u}{\partial x}(0,y) = 2 - y, \;\; u(1,y) = 2 - y; \;\; 0 \le y \le 1$$

Use the five point formula with $h = k = 1/3$.

3. Consider the following heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \;\; 0 \le x \le 1, \;\; t \ge 0$$
$$u(0,t) = u(1,t), \;\; \mathrm{t} \ge 0$$
$$u(x,0) = f(x), \;\; 0 \le x \le 1$$

Discuss the stability of a method that integrates time using the trapezoidal rule and approximates with a central difference scheme to solve the heat equation mentioned above. 8

Part – II (Marks: 8)

Answer ***any one*** question.

1. Find the solution of the integral equation

$$f(x) - \frac{1}{2}\int_0^1 (x+u) f(u)\, du = x$$

numerically by approximately the integral using Trapezoidal rule after dividing the interval [0, 1] into two equal parts. 8

2. The system of equations

$$x^2 + xy + y^2 = 7$$
$$x^3 + y^3 = 9$$

has a solution near $x = 1{\cdot}5$, $y = 0{\cdot}5$. Perform two iterations using Newton's Method to obtain approximate root.