Ref. No.: EX/SC/MATH/PG/CORE/TH/11/2024(S)

Master Of Science Examination - 2024

(Second Year, First Semester)

Mathematics

Core 11

(Integral Equation, Integral Transform and Calculus of variation)

Full Marks : 40 Time : 2 Hours

Symbols/Notations have their usual meaning

Use separate answerscripts for each part

Part I

(24 Marks)

1. Find the shortest path between the points (x_1, y_1) and (x_2, y_2). (8)

2. Answer any ONE (8)

a) Reduce the following boundary value problem to a Fredholm integral equation

$$y''(x) + y = 0,\ 0 \leq x \leq 1,\ y(0) = 0,\ y'(1) + 3y(1) = 1$$

b) Find the unique solution of

$$\phi(x) - \lambda \int_0^{\pi} \sin(x+t)\ \phi(t)dt = x, \quad 0 \leq x \leq \pi$$

3. Answer any ONE (8)

a) Find the Neumann series solution of

$$\phi(x) = 1 + \lambda \int_0^{\frac{\pi}{2}} \phi(t) \cos x dt \quad 0 \leq x \leq \pi/2.$$

b) Use Hilbert Schmidt theorem to solve the following integral equation

$$\phi(x) - \lambda \int_0^1 \min(x,t)\phi(t)\ dt = x, \quad 0 \leq x \leq 1.$$

[Turn over

PART II (Marks: 16)

(Integral Transforms)

Answer **any two** questions.

1. Use Fourier transform to solve the differential equation
$$\frac{d^2f}{dx^2} - f = \begin{cases} -1 & -1 < x < 1 \\ 0 & otherwise \end{cases}$$ [8]

2. (a) State and prove general Parseval relation for Fourier transform.
   (b) Find $\mathcal{H}_0[\frac{e^{-ar}}{r}]$, where $a > 0$. [4 + 4]

3. (a) Find $\mathcal{L}\{\int_0^t \frac{sinx}{x} dx\}$.
   (b) Using Laplace transform to solve the partial differential equation
   $\frac{\partial^2\phi}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 0,\ 0 \le x < \infty, \quad t \ge 0$
   with $\phi(0,t) = f(t)$, $\phi(x,t) \to 0$ as $x \to \infty$, $\phi(x,0) = 0$, $\frac{\partial\phi}{\partial t}(x,0) = 0$, $0 < x < \infty$.

[2 + 6]