BACHELOR OF SCIENCE EXAMINATION, 2024

(3rd Year, 1st Semester, Special Supplementary)

MATHEMATICS (Honours)

Paper - DSE - 2A

(Probability & Statistics)

Time : Two Hours **Full Marks : 40**

The figures in the margin indicate full marks.

Symbols / Notations and terminology have their usual meanings.

Answer **any eight** questions. $5 \times 8 = 40$

1. A point P is chosen at random on a circle of radius a and A is a fixed point on the circle. Find the probability that the chord AP will exceed the length of the side of an equilateral triangle inscribed in the circle.

2. The distribution function $F(x)$ of a random variable X is difined as follows:

$$\begin{aligned} F(x) &= A \text{ for } -\infty < x < -1 \\ &= B \text{ for } -1 \leq x < 0 \\ &= C \text{ for } 0 \leq x < 2 \\ &= D \text{ for } 2 \leq x < \infty \end{aligned}$$

where A, B, C and D are copnstants. Determine the values of A, B, C and D, it being given that $P(X = 0) = 1/6$ and $P(X > 1) = 2/3$.

3. The joint probability density function of continuous random variables X and Y is given by

$$f(x, y) = \begin{cases} kxy & 0 < x < 4 \;,\; 1 < y < 5 \\ 0 & elsewhere \end{cases}$$

Find the value of k and $P(X + Y < 2)$.

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4. If $E(X^2)$, $E(Y^2)$ and $E(XY)$ exist finitely for the two random variables X and Y, then prove that $[E(XY)]^2 \leq E(X^2)E(Y^2)$ and hence deduce that $-1 \leq \rho(X,Y) \leq 1$.

5. The random variables X and Y are connected by the linear relation $aX + bY + c = 0$. Prove that the correlation coefficient between X and Y is -1 if $ab > 0$.

6. If the random variables X and Y are independent then prove that $\rho(X,Y) = 0$. Give an example to show that $\rho(X,Y) = 0$ when X and Y are functionally dependent.

7. Write Characteristic function of a normal variate having mean m and standard deviation σ. Use the uniqueness theorem of Characteristic function to find the distribution of the sum of n mutually independent normal variates.

8. Express m_k as a function of a_0, a_1, $\cdots$, a_k.

9. Show that the sample mean is a consistent and unbiased estimate of population mean.

10. Let x_1, x_2, $\cdots$, a_6 be a sample of size 6 drawn from the population of $N(m, \sigma)$. Find the relation between a, b, m, σ such that the statistic

$$\sum_{i=2}^{6}(ax_i + bx_{i-1})^2$$

is an unbiased estimate of σ^2.

11. Find the maximum likelihood estimate of p for Binomial (N, p) population for a sample of size 100.

12. Find the confidence interval of m for a $N(m, \sigma)$ population when σ is known.

13. Describe the likelihood ratio testing method.

14. If $W = \{x : x \geq 1\}$ is the critical region for testing $H_0 : \theta = 2$ against the alternative $H_1 : \theta = 1$ on the basis of a single observation x from the population having probability density function

$$f(x) = \theta e^{-\theta x}, 0 \leq x < \infty$$

for the unknown parameter θ, obtain the power of the test.