B. Sc. Mathematics Examination, 2024

(3rd Year, 1st Semester)

Mathematics (Core)

Paper : Core/Math/Th/11

(Partial Differential Equation)

Time : *Two Hours* *Full Marks* : 40

(*20 marks for each Group*)

Symbols/Notations have their usual meanings.

Use separate Answer scripts for each Group.

GROUP—A

(20 Marks)

Answer ***any four*** questions

All questions carry equal marks

1. Given a surface of the form $F(u,v)=0$ where $u=u(x, y, z)$ and $v=v(x, y, z)$ are known functions of x, y, z ; $z=z(x, y)$ and F is an arbitrary function of u and v having first order derivatives with respect to u and v. Find the partial differential equation satisfied by the surface $F(u, v)=0$. 5

2. Find the equation of the surface satisfying $4yzp + q + 2y = 0$ and passing through $y^2 + z^2 = 1$, $x + z = 2$. 5

3. Find the solution of the equation $(y + 2zx)\,p - (x + 2zy)\,q = \frac{1}{2}(x^2 - y^2)$, with the Cauchy data $z = 0$ on $x - y = 0$. 5

4. (*a*) Using the method of separation of variables suitably, find the solution of the Cauchy problem $z_x + y^2 + z_y^2 = 0$. 3

(*b*) Reduce the equation $z_x + 2xy\, z_y = x$ to canonical form. 2

5. Consider the first order quasilinear partial differential equation given by $P(x\, y, z)\, p + Q(x, y, z)\, q = R(x, y, z)$, where $P(x\, y, z)$, $Q(x, y, z)$, $R(x, y, z)$ are known functions of x, y, z. Under what condition does there exist an unique solution $z(x, y)$ of the given partial differential equation which assumes a prescribed value on a given regular arc γ in xy plane? 5

6. Use Charpit's method to solve $px + qy = z(1 + pq)^{1/2} = 0$. 5

SECTION—B

(20 Marks)

Answer ***any two*** questions

1. Classify and transform the canonical form of the equation

$$\sin^2 x\, u_{xx} + \sin(2x)\, u_{xy} + \cos^2 x\, u_{yy} = x$$ 10

2. Obtain the solution of the initial-value problem given below :

$$\begin{aligned} u_{tt} - c^2 u_{xx} &= \sin(kx - \omega t), -\infty < x < \infty \\ u(x, 0) &= u_t(x, 0) = 0 \text{ for all } x \in \mathbb{R} \end{aligned}$$

where c, k and ω are constants. Discuss the cases $\omega \neq ck$ and $\omega = ck$. 10

3. Use the method of separation of variables to solve

$$\begin{aligned} u_{tt} &= c^2 u_{xx}, 0 < x < \pi, t > 0, \\ u(x, 0) &= \cos x, u_t(x, 0) = 0, 0 \le x \le \pi \\ u_x(0, t) &= 0, u_x(\pi, t) = 0, t > 0. \end{aligned}$$ 10

★ ★ ★