4. *(a)* Use dominance principle to reduce the following payoff matrix and then solve it :

$$\begin{pmatrix} 4 & 2 & 0 & 2 & 1 & 1 \\ 4 & 3 & 1 & 3 & 2 & 2 \\ 4 & 3 & 7 & -5 & 1 & 2 \\ 4 & 3 & 4 & -1 & 2 & 2 \\ 4 & 3 & 3 & -2 & 2 & 2 \end{pmatrix}$$

(b) Solve graphically the game whose payoff matrix is given below :

Player B

		B_1	B_2
Player A	A_1	3	−4
	A_2	2	5
	A_3	−2	8

5+5

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B. SC. MATHEMATICS (HONS.) EXAMINATION, 2024

(3rd Year, 1st Semester)

LINEAR PROGRAMMING & GAME THEORY

PAPER : DSE-1A

Time : *Two Hours* *Full Marks* : 40

(*20 marks for each Group*)

Symbols/Notations have their usual meanings.

Use separate Answer scripts for each Group.

GROUP—A

(20 Marks)

Answer ***any two*** questions

All questions carry equal marks

1. *(a)* $x_1 = 1,\ x_2 = 2,\ x_3 = 1,\ x_4 = 0$ is a feasible solution of the set of equations

$$11x_1 + 2x_2 - 9x_3 + 4x_4 = 6$$
$$15x_1 + 3x_2 - 12x_3 + 5x_4 = 9$$

Find out the basic feasible solutions.

(b) Solve the following linear programming problem by simplex method :

Maximize $Z = 2x_1 - 3x_2$

subject to

$$-x_1 + x_2 \geq -2$$
$$5x_1 + 4x_2 \leq 46$$
$$7x_1 + 2x_2 \geq 32$$
$$x_1, x_2, x_3 \geq 0$$

5+5

2. *(a)* Prove that $x = \{(x_1, x_2) \mid x_1 + x_2 \leq 50, x_1 + 2x_2 \leq 80, 2x_1 + x_2 \geq 20, x_1, x_2 \geq 0\}$ is a convex set.

(b) Solve the following linear programming problem by two phase method :

Minimize $Z = x_1 + x_2$

subject to

$$3x_1 + 2x_2 \geq 30$$
$$2x_1 + 3x_2 \geq 30$$
$$x_1 + x_2 \leq 5$$
$$x_1, x_2 \geq 0$$

4+6

3. *(a)* Find the dual of the following linear programming problem :

Maximize $Z = x_1 - x_2 + 3x_3 + 2x_4$

subject to

$$x_1 + x_2 \geq -1$$
$$| x_1 - 3x_2 - x_3 | \leq 7$$
$$x_1 + x_2 - 3x_3 = -2$$

To

		I	II	III	IV	a_i
From	A	16	10	17	18	2
	B	15	13	12	13	6
	C	12	17	20	11	7
	b_j	3	3	4	5	

Solve the problem using transportation algorithm for minimum cost. 4+6

3. *(a)* Show that in a game with 2×2 payoff matrix if two elements in a row or column be equal, then the game has a saddle point.

Suppose that A is paid ₹ 8 if two coins turn both head and ₹ 1 if both coins turn tails. B is paid ₹ 3 when the two coins do not match. Given the choice of being A or B, which one would you choose and what would be your strategy?

(b) Two companies A and B are competing for the same product. Their different strategies are given by the following payoff matrix :

Company A

Company B	A_1	A_2	A_3
B_1	2	−2	3
B_2	−3	5	−1

Use linear programming to determine the best strategies for the both the players. 4+6

(b) Use duality to solve the following linear programming problem :

Minimize $Z = x_1 + 2x_2 + 3x_3$

subject to

$$x_1 - x_2 + x_3 \geq 4$$

$$x_1 + x_2 + 2x_3 \leq 8$$

$$x_2 - x_3 \geq 2$$

$$x_1, x_2, x_3 \geq 0$$ 4+6

GROUP—B

(20 Marks)

Answer ***any two*** questions

All questions carry equal marks

1. *(a)* Derive a mathematical model for a cost minimizing transportation problem. Show that every transportation problem has a feasible solution.

(b) A city corporation has decided to carry out road repairs on main four arteries of the city. The government has agreed to make a special grant of ₹50 lakhs towards the cost with a condition that the repairs must be done at the lowest cost and quickest time. If conditions warrant, then a supplementary token grant will also be considered favourably. The corporation has floated tenders and 5 contractors have sent in their bids. In order to expedite work, one road will be awarded to only one contractor.

Cost of repair (₹ lakhs)

Contractors	R_1	R_2	R_3	R_4
C_1	9	14	19	15
C_2	7	17	20	19
C_3	9	18	21	18
C_4	10	12	18	19
C_5	10	15	21	16

(i) Find the best way of assigning the repair work to the contractors and the cost.

(ii) If it is necessary to seek supplementary grants, then what amount should be sought? 4+6

2. *(a)* If $b_{ij} = c_{ij} - u_i - v_j \ (i, j = 1, 2, \ldots, n)$ where u_i and v_j are constants, then show that an optimal solution of the assignment problem with cost matrix $B = (b_{ij})$ is also an optimal solution of the assignment problem with cost matrix $C = (c_{ij})$.

(b) Hindustan construction company needs 3, 3, 4 and 5 million cubic feet of fill at four earthen dam-sites in Punjab. It can transfer the fill from three mounds *A*, *B* and *C* where 2, 6 and 7 million cubic feet of fill is available respectively. Costs of transporting one million cubic feet of fill from mounds to the four sites in lakhs are given in the table.