Ref. No.: SC/MATH/UG/CORE/TH/11/2024(S)

Bachelor Of Science Examination - 2024

(Third Year, First Semester)

Mathematics

Core 11

(Partial Differential Equation)

Full Marks : 40 Time : 2 Hours

Symbols/Notations have their usual meaning

Use separate answerscripts for each part

Part I

(20 Marks)

Answer any FOUR questions

All questions carry equal marks

1. Use the method of separation of variables to find the solution of
$z_x - z_y = z, \quad z(x,0) = 4e^{-3x}$. 5

2. Find the equation of the surface satisfying $yz_x + xz_y = x^2 + y^2$ and passing through the curve $x = 1 - t,\ y = 1 + t,\ z = 1 + t^2$. 5

3. Find the solution of the equation $x(y^2 + z)z_x - y(x^2 + z)z_y = (x^2 - y^2)z$, with the Cauchy data $z = 1$ on $x + y = 0$. 5

4. Reduce to canonical form and find the general solution of $z_x - yz_y - z = 1$. 5

5. 2. A surface is given by $ax + by + cz = \phi(x^2 + y^2 + z^2)$ where ϕ is an arbitrary function, $a,\ b,\ c$ are known parameters and $z = z(x,y)$. Form the partial differential equation. 5

6. Find the complete solution and singular solution of $x^2p^2 + y^2q^2 = z$. 5

Ref. No.: Ex/SC/MATH/UG/CORE/TH/11/2024(S)

B.Sc Maths 3rd Year 1st Sem Special Supple Exam 2024

Partial Differential Equations

Full Marks:40 Time: Two Hours

Part-II
(20 Marks)

1. (a) (CO4) (3)
Prove that the Laplacian of the gravitation potential is zero.

(b) (CO2) (7)
Reduce the canonical form of the partial differential equation

$$3u_{xx} + 10u_{xy} + 3u_{yy} = 0.$$

Hence find the general solution.

2. (CO3) (10)
Solve the intial-boundary value problem

$$u_{tt} - c^2 u_{xx} = 0, \quad t > 0, \ 0 < x < \infty$$

Subject to

$$\begin{aligned} u(x,0) &= f(x), \quad 0 \le x < \infty \\ u_t(x,0) &= g(x), \quad 0 \le x < \infty \\ u_x(0,t) &= 0, \quad t \ge 0 \end{aligned}$$

Derive the conditions on f and g for the existence of solution.

3. (CO4) (10)
Use the method of separation of variables to solve

$$u_t - u_{xx} = 0, \quad t > 0, \ 0 < x < \pi$$

Subject to

$$\begin{aligned} u(x,0) &= 2\cos^2 x, \quad 0 \le x \le \pi \\ u_x(0,t) &= u_x(\pi,t) = 0, \quad t \ge 0 \end{aligned}$$