# MASTER OF SCIENCE EXAMINATION, 2018

**(1st Year, 1st Semester)**

**MATHEMATICS**

**Unit - 1.4**

**(General Mechanics)**

Full Marks : 50 Time : Two Hours

*The figures in the margin indicate full marks.*

Notations/Symbols have their usual meanings.

Answer any *five* questions.

1. (a) Derive Lagrange's equation of motion for a conservative and unconnected holonomic system.

(b) Show that the K.E. of a moving system can be expressed as

$$T = \frac{1}{2}\sum_{i,j} a_{i},\ \dot{q}_i\, \dot{q}_j + \sum_i b_i\, \dot{q}_i + c$$

State the expressions for the coefficients. 7+3

[*Turn over*]

2. (a) Define generalized momentum corresponding to a generalized co-ordinate. Show that generalized momentum is conserved corresponding to a cyclic co-ordinate. Write down Lagrange's equation of motion as the evolution of generalized momentum.

(b) The K.E. and the P.E. of a dynamical system is given by

$$2T = \left(q_1^2 + q_2^2\right)\left(\dot{q}_1^2 + \dot{q}_2^2\right) \text{ and } V = \frac{1}{\left(q_1^2 + q_2^2\right)}$$

By solving using Liouville's theorem, show that $q_1$ and $q_2$ are related as $a^2 q_1^2 + b^2 q_2^2 + 2ab\, q_1 q_2 \cos r = \sin^2 r$ where $a$, $b$, $r$ are constants. (2+2+1)+5

3. (a) Define action of a mechanical system. State and prove the principle of least action.

(b) If the K.E. $T(q,\ \dot{q})$ of a system is a homogeneous quadratic in the velocity $\dot{q}$'s i.e.

$$T(q,\ \dot{q}) = \frac{1}{2}\sum a_i,\ \dot{q}_i\, \dot{q}_j$$

[*Turn over*]

and $T'(q,\ p)$ is what $T$ becomes when expressed in terms of variables $q$ and $p$, then prove that

$$2T' + \frac{D}{\Delta} = 0$$

where $\Delta = \det(a_{ij})$ and $D = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} & p_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & p_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} & p_n \\ p_1 & p_2 & \cdots & p_n & 0 \end{vmatrix}$.

(2+4)+4

4. (a) Write down Hamilton's canonical equations in symmetrical form.

(b) If $A$ and $B$ are two constants of motion, then show that their poisson bracket is also a constant of motion.

(c) State the interrelation between Poisson bracket and Lagrange's bracket.

(d) Solve the one dimensional harmonic oscillator

$$T = \frac{1}{2}\dot{q}^2, \quad V = \frac{1}{2}\mu^2 q^2$$

[*Turn over*]

using canonical transformation $(q,\ p) \to (Q,\ P)$ with the generating function $F_1 = \frac{1}{2}\mu q^2 \cot Q$. 2+3+2+3

5. (a) Derive D'Alembert's principle from Hamilton's principle.

(b) State principle of linear and angular momentum.

(c) Show that $J_1 = \int\limits_{S_2} \sum\limits_{1} dq_i\, dp_i$, is invariant under canonical transformation. 3+3+4

6. (a) Derive Euler's dynamical equations.

(b) A particle is revolving on a smooth plane about a centre of force, the acceleration is proportional to the distance (i.e. $\mu r$) and when the body arrives at an apse the plane begins to revolve with an angular velocity $\frac{1}{2}\sqrt{3\mu}$ about the apsidal line. Show that the subsequent orbit described on the plane will be a portion of a parabola and that when the particle leaves the plane its velocity will be $\sqrt{3}$ times the velocity at the vertex of the parabola. 4+6

7. (a) What do you mean by normal co-ordinates ? Show that the time-periods of small oscillation of a constraint system

[*Turn over*]

will lie between the time periods of the unconstraint system.

(b) Two uniform rods of same mass $m$ and of same length $2a$ are freely jointed at a common extremity and rests upon two smooth pegs which are in the same horizontal plane so that each rod is inclined at same angle $\alpha$ to the vertical. Show that the time of small oscillation when the joint moves in a vertical straight line through the centre of the line joining the pegs is

$$2\pi\sqrt{\frac{a}{9g}\frac{\left(1+3\cos^2\alpha\right)}{\cos\alpha}}.$$

2+4+4

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