Ex/M.Sc./Math/12/2.4/133/2018

MASTER OF SCIENCE EXAMINATION, 2018

[1st Year, 2nd Semester]

MATHEMATICS

Unit - 2.4

(Mechanics of Continua)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

Symbols have their usual meanings.

Answer any *five* of the following questions.

1. (a) Derive the Eulerian strain-tensor e_{ij} in terms of the displacement components of a material point in the deformed continuum body.

   (b) Show that the necessary and sufficient condition for rigid body deformation at each point is $e_{ij} = 0$. 4+6

2. (a) Determine the strain invariants from infinitesimal strain-tensor $\left[\varepsilon_{ij}\right] = \begin{bmatrix} 5 & -1 & -1 \\ -1 & 4 & 0 \\ -1 & 0 & 4 \end{bmatrix}$.

[*Turn over*]

Also determine the principal strains. Obtain the strain invariants from principal strains and show the equivalence of those strain invariants.

(b) Prove that the stress vector at a point on any arbitrary plane surface is a linear function of three stress vectors acting on any three mutually perpendicular planes through that point. 5+5

3. Discuss the Mohr's circle diagram in the stress plane for the cases when (i) $\tau_{11} \neq \tau_{22} \neq \tau_{33}$, (ii) $\tau_{11} = \tau_{22} \neq \tau_{33}$ and (iii) $\tau_{11} = \tau_{22} = \tau_{33}$. Also determine the normal stress corresponding to the maximum shearing stress via Mohr's circle diagram. 7+3

4. (a) The state of stress throughout a continuum is given with respect to the Cartesian axes $0x_1x_2x_3$ by

$$\begin{bmatrix} 3x_1x_2 & 5x_2^2 & 0 \\ 5x_2^2 & 0 & 2x_3 \\ 0 & 2x_3 & 0 \end{bmatrix}$$

Compute the stress vector and the magnitude of the normal stress acting at the point $P\left(2,1,\sqrt{3}\right)$ of the plane that is tangent to the cylindrical surface $x_2^2 + x_3^2 = 4$ at P.

[*Turn over*]

(b) Show that for linearly elastic isotropic material, the stress-strain relation can be put mathematically as

$$\tau_{ij} = \frac{vE}{(1+v)(1-2v)}\varepsilon_{kk}\delta_{ij} + \frac{E}{1+V}\varepsilon_{ij},$$

where E is the Young's modulus and v is the Poisson ratio. 4+6

5. (a) Write a short note on material derivative.

(b) State Reynold's transport theorem.

(c) Derive Euler's equation of motion for perfect fluid. 3+2+5

6. Discuss the plane Couette flow of Newtonian viscous fluid. hence determine the volumetric flow rate, average velocity and wall shear stress at both the walls of the channel. 6+2+2

7. State conservation law of energy and hence derive thermal energy equation based on the Fourier's law of heat conduction in the form of

$$\rho\frac{D\theta}{Dt} = k\nabla^2\theta + \sigma_{ij}v_{ij},$$

Also deduce the thermal energy equation for solid body material. 2+6+2

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