MASTER OF SCIENCE EXAMINATION, 2018

(2nd Year, 2nd Semester)

MATHEMATICS

Unit - 4.3 (A-2.4)

[Fluid Mechanics-II]

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

Symbols/Notations used have their usual meaning.

Answer question no. 6 and any *three* from the rest.

02+16×3=50

1. Assuming the relation

$$\frac{p_+}{p_-} = \frac{(\gamma+1)\rho_+ - (\gamma-1)\rho_-}{(\gamma+1)\rho_- - (\gamma-1)\rho_+} ,$$

State and prove Zemplen's theorem. Here p_+ , ρ_+ and p_- , ρ_- are the pressure and density of the gas after and before crossing the discontinuity surface. 16

[*Turn over*]

2. Establish the hodograph equations $\phi_\beta = \frac{\rho_0 q}{\rho}\psi_q$.

   $\phi_q = \frac{M^2 - 1}{\rho q}\psi_\beta$ for two dimensional irrotational motion of a compressible fluid. Hence derive Chaplygin's equation for the stream function for a steady two dimensional isentropic flow of a compressible fluid. 16

3. Derive the equation of a shock polar in case of oblique shock. 16

4. Knowing that the characteristics of all steady two dimensional irrotational motions in the hodograph plane are fixed curves, determine the velocity in the region of physical plane bounded by an arc *AB* of certain curve not being a characteristic in physical plane and arcs *AC* and *BC* which are characteristics of two families through *A* and *B*. The velocity field on arc *AB* is known. 16

5. Write down the equation for perturbed stream function for a supersonic gas flow past a thin profile. Show that the angle formed by the perturbation lines with the direction of unperturbed motion is given by $\alpha = \arcsin \frac{1}{M_\infty}$, M_∞ being Mach number for unperturbed stream. Show that the

[*Turn over*]

longitudinal and transverse components of perturbation velocity are proportional to the local angle of inclination of the streamlines of the perturbed motion to the direction of unperturbed flow. 16

6. State second law of thermodynamics. 2