# BACHELOR OF SCIENCE EXAMINATION, 2018

**(1st Year, 2nd Semester)**

**MATHEMATICS**

**Paper - 6S**

**(Vector Algebra)**

**[Old Syllabus]**

Full Marks : 50 Time : Two Hours

*The figures in the margin indicate full marks.*

( Symbols and Notations have their usual meanings)

Answer any *five* questions.

( Use vector approach to solve all problems)

( $\hat{i}$, $\hat{j}$ and $\hat{k}$ are the unit vectors in the positive directions of $x$, $y$ and $z$ axis respectively)

1. (a) Suppose ABCD is a quadrilateral. The diagonals AC and BD meet at P. The sides AB, DC meet at Q and BC, AD meet at R. P divides AC in the ratio 1:2. Find the ratio in which Q divides AB and CD, R divides BC and AD.

[*Turn over*]

(b) Show that the three points whose position vectors are $A(-2,3,5)$; $B(1,2,3)$; $C(7,0,-1)$ lie on a line.

(c) Three points A, B, C are given by $(5,-3,-4)$, $(2,-3,-1)$ and $(2,-3,3)$. Find their centroid. Also find the centroid if masses 4, 5, 3 respectively be associated with them. 4+2+4

2. (a) Prove that the internal bisectors of the three angles of a triangle are concurrent.

(b) ABCDEF is a regular hexagon; $\overrightarrow{AB}=\vec{a}$, $\overrightarrow{BC}=\vec{b}$. Express the remaining sides and diagonals in terms of $\vec{a}$ and $\vec{b}$.

(c) Prove that the vectors $\vec{A}=\hat{i}-3\hat{j}+2\hat{k}$, $\vec{B}=2\hat{i}-4\hat{j}-\hat{k}$, $\vec{C}=3\hat{i}+2\hat{j}-\hat{k}$ are linearly independent. 4+4+2

3. (a) Prove that perpendicular bisectors of the sides of a triangle are concurrent.

(b) Using vector product prove that

$$\cos(A-B)=\cos(A)\cos(B)+\sin(A)\sin(B)$$

(c) Prove geometrically $\vec{a}\cdot\left(\vec{b}+\vec{c}\right)=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$. 4+3+3

4. (a) Express the area of a triangle in terms of the position vectors of its vertices. Hence find the area of a triangle with vertices $\vec{a} = (2, -1, 1), \vec{b} = (3, 0, 1)$ and $\vec{c} = (1, -2, 3)$.

(b) In a triangle ABC, prove that

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$, where $a$, $b$, $c$ are the lengths of the sides opposite to $A$, $B$, $C$ respectively.

(c) Prove that a necessary and sufficient condition for the three vectors $\vec{A}, \vec{B}$ and $\vec{C}$ to be coplanar is that $\vec{A} \cdot (\vec{B} \times \vec{C}) = 0$. 4+3+3

5. (a) Find the vector equation of the straight line passing through the point $(-1, 1, -3)$ and perpendicular to the line $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, where $\mathbf{a} = (3, -1, 2)$ and $\mathbf{b} = (-2, 3, -4)$.

(b) Let the position vectors of two points $P$ and $Q$ are given by $\mathbf{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\mathbf{b} = \hat{i} - 2\hat{j} - 4\hat{k}$. Find an equation for the plane passing through $Q$ and perpendicular to the line $PQ$. What is the distance of the plane from the point $(-1, 1, 1)$? 4+6

[*Turn over*]

6. (a) Find the image of the point $(-3,8,4)$ in the plane $\mathbf{a}.\mathbf{r}+1=0$, where $\mathbf{a}=(6,-3,-2)$.

(b) Find the shortest distance from $(6,-4,4)$ to the line joining (2,1,2) and (3,–1,4). 5+5

7. (a) Write the vector equation of the sphere passing through the point (2,3,1) and the circle $\mathbf{r}.\mathbf{r}+2\mathbf{a}.\mathbf{r}+4=0$, $\mathbf{b}\cdot\mathbf{r}=6$, where $\mathbf{a}=(3,-4,-2)$ and $\mathbf{b}=(1,2,3)$.

(b) Prove that the plane $\mathbf{a}.\mathbf{r}=12$, where $\mathbf{a}=(2,1,-1)$ touches the sphere $\mathbf{r}.\mathbf{r}=24$. Find its point of contact.

5+5