Bachelor of Science Examination, 2018

(1st Year, 2nd Semester)

MATHEMATICS -I

Paper - GE-2

[New Syllabus]

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

Use a separate Answer sheet for each Group.

Symbols and Notations have their usual meanings.

Part - I

(15 marks)

Answer any *five* questions. 5×3=15

1. State Leibnitz theorem on successive Differentiation. Using this theorem prove that if $y = \cos\left(m \sin^{-1} x\right)$, then $\left(1-x^2\right)y_{n+2} - (2n+1)xy_{n+1} + \left(m^2 - n^2\right)y_n = 0$. Also find y_n for $x = 0$.

[*Turn over*]

2. Evaluate $\lim\limits_{x \to 0} \left(\dfrac{\tan x}{x} \right)^{1/x^2}$

3. Examine the continuity of the function

$$f(x, y) = \begin{cases} \dfrac{x^4 + y^4}{x - y} & , \text{ when } x \neq y \\ 0 & , \text{ when } x = y \end{cases}$$

at (0, 0).

4. State Euler's theorem for homogeneous function of two variables : If $V = \sin^{-1} \sqrt{\dfrac{x^{1/3} + y^{1/3}}{x^{1/2} + y^{1/2}}}$, then prove that $x^2 \dfrac{\partial^2 v}{\partial x^2} + 2xy \dfrac{\partial^2 v}{\partial x \partial y} + y^2 \dfrac{\partial^2 v}{\partial y^2}$

$$= \frac{\tan v}{12} \left(\frac{13}{12} + \frac{\tan^2 v}{12} \right).$$

5. Let $f(x, y) = \begin{cases} x \sin \dfrac{1}{y} + y \sin \dfrac{1}{x} & , \text{ when } xy \neq 0 \\ 0 & , \text{ when } xy = 0 \end{cases}$

then show that $\lim\limits_{(x,y) \to (0,0)} f(x, y) = 0$ but repeated limits do not exist.

Part - II

(20 marks)

Answer any *two* questions.

6\. (a) If the equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two straight lines equidistant from the origin, show that $f^4 - g^4 = c\left(bf^2 - ag^2\right)$.

(b) Show that the equation of the chord joining the points $\theta = \theta_1$ and $\theta = \theta_2$ on the circle $r = 2a\cos\theta$ is $r = \cos\left(\theta - \theta_1 - \theta_2\right) = 2a\cos\theta_1\cos\theta_2$. 5+5

7\. (a) A plane passes through a fixed point (a,b,c) and cuts the axes at A, B, C. Show that locus of the centre of the sphere OABC is $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$.

(b) Any tangent to an ellipse with centre C meets the director circle in P and D. Prove that CP and CD are in the directions of the conjugate diameters of the ellipse.

(c) Find the equation of the circle cutting $x^2 + y^2 + 2x - 9 = 0$ and $x^2 + y^2 - 8x - 9 = 0$ orthogonally and touching $y = x + 4$. 3+3+4

[*Turn over*]

8. (a) Calculate the polar of a point $\left(x_1, y_1\right)$ with respect to the circle $x^2+y^2+2gx+2fy+c=0$.

(b) Find the necessary and sufficient conditions that the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ may represent a pair of straight lines. 5+5

Part - III

(15 marks)

Answer any *three* questions. 5×3=15

9. (a) State Fundamental theorem of integral calculus.

(b) Show that $\frac{1}{2}<\int_0^1 \frac{dx}{\sqrt{4-x^2+x^3}}<\frac{\pi}{6}$. 2+3

10. Test the convergence of the following :

(i) $\int_{-\infty}^{\infty} \frac{x\,dx}{x^4+1}$

(ii) $\int_0^{\infty} x^2 e^{-x} dx$ 3+2

[*Turn over*]

11. (a) Show that $B(m,n) = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}}\,dx$

(b) Evaluate $\int_0^{\frac{\pi}{2}} \sin^4\theta \,\cos^6\theta \,d\theta$. 2+3

12. Evaluate $\iint_R \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right) dx\,dy$, where R consists of points in the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 5

13. Evaluate $\iint_R \left(x^2 + y^2\right) dx\,dy$ over the region R enclosed by the triangle having its vertices at (0, 0), (1, 0) and (1, 1). 5