# Bachelor of Science Examination, 2018

## (1st Year, 2nd Semester)

## MATHEMATICS (Honours)

## Paper - 2.3

## (Algebra - II)

## [ Old Syllabus ]

Full Marks : 50 Time : Two Hours

*The figures in the margin indicate full marks.*

Notations and Symbols have their usual meanings.

Answer any *five* questions. 10×5=50

1. (a) (i) Let $A$ and $B$ be two sets such that $|A| = m$ and $|B| = n$. Find the total number of non-constant functions from $A$ into $B$.

(ii) Let $A$ and $B$ be two sets such that $|A| = 5$ and $|B| = 2$. Find total number of surjective mappings from $A$ onto $B$. 3+2

(b) State the fundamental theorem of arithmetic. Show that the number of primes is infinite. 2+3

[*Turn over*]

2. (a) Define group. Let $G = \left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} : a(\neq 0) \in \mathbb{R} \right\}$. Does $G$ form a group *w.r.t.* matrix multiplication ? Justify your answer. 2+3

(b) Define centre of a group. Show that the centre of a group $G$ is a subgroup of $G$. Let $G$ be a group and $Z(G)$ be the centre of $G$. If $a$ is the only element of order 2 in $G$, show that $a \in Z(G)$. 1+2+2

3. (a) Define cyclic group. Show that $(\mathbb{Q}+)$ is not a cyclic group. Hence conclude that $(\mathbb{R}+)$ is not a cyclic group. 1+2+2

(b) State and prove Lagrange's theorem of group. 2+3

4. (a) State first isomorphism theorem of group. By using this theorem, conclude that any epimorphism from $(\mathbb{Z},+)$ onto itself is an isomorphism. 2+3

(b) Define $GL(n,\mathbb{R})$ and $SL(n,\mathbb{R})$. Show that

$$GL(n,\mathbb{R}) \big/ SL(n,\mathbb{R}) \simeq \mathbb{R}^*, \text{ where } \mathbb{R}^* = \mathbb{R} \setminus \{0\}.$$

2+3

[*Turn over*]

5. (a) Define ring. Show that any ring of order 6 is commutative. 2+3

(b) Define characteristic of a ring. Give an example of an infinite ring whose characteristic is finite. Show that the characteristic of an integral domain is either zero or prime. 1+2+2

6. (a) Define Boolean ring. Show that every Boolean ring has characteristic 2 and also commutative. 1+2+2

(b) Define field. Show that every finite integral domain is a field. Give an example of an infinite integral domain which is not a field. 1+3+1

7. (a) State De Moivres theorem. If $\alpha$ and $\beta$ are the roots of the equation $x^2 - 2x\cos\theta + 1 = 0$ then find the equation whose roots are $\alpha^n$ and $\beta^n$, where $n$ is a positive integer. 2+3

(b) Find the principal value of $(1+i)^i$. Show that $\sin\left(i\log\frac{a-ib}{a+ib}\right) = \frac{2ab}{a^2+b^2}$, where $a$, $b$ are reals.

2+3