BACHELOR OF SCIENCE EXAMINATION, 2018

(1st Year, 2nd Semester)

MATHEMATICS (Subsidiary)

Paper - 5S

[Algebra - II]

[Old Syllabus]

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

(Symbols and Notations have their usual meanings)

Answer any *five* questions.

1. (a) Solve, if possible the system of equations

$$\begin{aligned} x+4y &= 2 \\ 2x+7y &= 5 \\ 4x+9y &= 15 \end{aligned}$$

(b) Obtain the fully reduced normal form of the matrix and hence find the rank of

$$\begin{pmatrix} 0 & 0 & 1 & 2 & 1 \\ 1 & 3 & 1 & 0 & 3 \\ 2 & 6 & 4 & 2 & 8 \\ 3 & 9 & 4 & 2 & 10 \end{pmatrix}$$

5+5

[*Turn over*]

2. (a) Find the eigen values and the corresponding eigen vectors of the matrix.

$$\begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

(b) If A is a real orthogonal matrix and $(I+A)$ is non singular then prove that $(I+A)^{-1}(I-A)$ is skew symmetric. 6+4

3. (a) Expand by Laplace's Method to prove that

$$\begin{vmatrix} 0 & a & b & c \\ -a & 0 & d & e \\ -b & -d & 0 & f \\ -c & -e & -f & 0 \end{vmatrix} = (af - be + cd)^2$$

(b) Investigate for what values of λ and μ, the following equations

$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 10 \\ x + 2y + \lambda z &= \mu \end{aligned}$$

have (i) no solution, (ii) a unique solution, (iii) an infinite number of solutions. 4+6

[*Turn over*]

4. (a) State Cayley-Hamilton theorem. Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ 2 & 1 & 4 \end{bmatrix}.$$

Hence find A^{-1}.

(b) If A and P are $n \times n$ matrices and P is non singular then prove that A and $P^{-1}AP$ have the same eigen values.

6+4

5. (a) Let ρ be an equivalence relation on the set A. Then prove that for all $a, b \in A$, either $[a] = [b]$ or $[a] \cap [b] = \varphi$, where $[a]$ denotes ρ-equivalence class of a.

(b) Determine whether the permutatons are even or odd.

(i) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 1 & 3 & 5 & 4 & 7 & 6 & 8 \end{pmatrix}$,

(ii) (1 2 3 8) (3 5 8) (1 3). 6+4

[*Turn over*]

6. (a) If A, B, C are any three subsets of a set X, then prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

(b) Prove that every cyclic group is an abelian group. Is an abelian group necessarily a cyclic group ? Illustrate with an example. 4+6

7. (a) Prove that $\mathbb{Z}_4$, the classes of residues of integers modulo 4, forms an abelian group with respect to '+', addition (modulo 4).

(b) Describe all the permutations on the set $\{1, 2, 3\}$ and find their respective orders. 5+5