BACHELOR OF SCIENCE EXAMINATION, 2018

(3rd Year, 2nd Semester)

MATHEMATICS (Honours)

Paper - 6.1

(Probability Theory)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

Symbols / Notations have their usual meanings.

Answer any *five* questions.

1. (a) Sate and prove Bayes theorem. 4

   (b) Three urns contain respectively 1 white and 2 black balls ; 2 white and 1 black balls ; 2 white and 2 black balls. One ball is transferred from the first to the second urn ; then one ball is transferred from second to third urn. Find the probability that the ball is white. 4

   (c) A box of fuses contains 20 fuses, of which 5 are defective. If 3 of the fuses are selected at random and removed from the box in succession without

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replacement, what is the probability that all fuses are defective ? 2

2. (a) If $F(x)$ is the cumulative distribution function of a random variable X, then prove that (i) $F(x)$ is monotonic non-decreasing ; (ii) $F(-\infty) = 0$; (iii) $F(\infty) = 1$. 4

(b) Five balls are drawn from an urn containing 4 white and 6 black balls. Find the probability distribution of the number of white balls drawn when the balls are drawn (i) with replacement ; (ii) without replacement. 4

(c) The numbers 1, 2, ...n are arranged in random. What is the probability that the numbers 1 and 2 are always together ? 2

3. (a) Prove that limiting form of binomial (n, p) distribution with $n \rightarrow \propto$, $p \rightarrow 0$, while np remains constant, is the Poisson distribution with parameter $\lambda = np$. 3

(b) Show that a function which is $|x|$ in $(-1, 1)$ and zero elsewhere is a possible probability density function, and find the corresponding distribution function. 4

(c) A point is chosen at random on a semi-circle having centre at the origin and radius a and projected on the

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diameter. Find the density function of the distance of the point of projection from the centre. 3

4. (a) Find mean and variance of binomial (n, p) distribution. 4

(b) For normals $(u, \quad)$ distribution prove that $u_k = (k \quad 1) \quad ^2 \quad _{k-2}$, where u_k denotes kth central moment. Hence prove that $u_{2k+1} \quad 0$ and $u_{2k} = 1.3.5......(2k \quad 1) \quad ^{2k}$. 4

(c) Find coefficient of skewness for the following data : $f(1) = 0.05$, $f(2) = 0.20$, $f(2) = 0.25$, $f(4) = 0.50$, $f(5) = 0.10$ and $f(6) = 0.05$. 2

5. (a) The joint probability density function of two variates X, Y is given by

$$f(x, y) = (6 - x - y)/8, \; 0 \quad x \quad 2, \; 2 \quad y \quad 4$$
$$= 0 \qquad \text{elsewhere}$$

Calculate $P(X < 1, Y \quad 3)$, $P(X + Y \quad 3)$, $P(X < 1/Y \quad 3)$ and $P(X < 1/Y \quad 3)$. 5

[*Turn over*]

(b) Two points are independently chosen at random in the interval (0, 1). Find the probability that the distance between them is less than a fixed number $k\,(0 < k \quad 1)$. 3

(c) Find the moment generating function of the standard normal distribution. 2

6. (a) Find correlation coefficient for normal bivariate distribution. 4

(b) If for any pair of linearly dependent random variables X, Y we set

$$U = X\cos\alpha + Y\sin \quad , V \quad Y\cos \qquad X\sin \quad ,$$

(α, a constant), then prove that V has a one point distribution if $\tan\alpha = \rho\dfrac{\sigma_y}{\sigma_x}$. 2

(c) Let the random variable X and Y have the joint density function

$$f(x, y) = 1/4, \text{ if } (x, y) \in \{(0,1),(0-1),(1,0)(-1,0)\}$$
$$= 0 \quad \text{otherwise}$$

What is the covariance of X and Y? Are the random variables X and Y independent? 4

[*Turn over*]

7. (a) State and prove central limit theorem. 4

(b) If the probability density of X is given by

$$f(x) = \begin{cases} 630x^4(1-x)^4, & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability that it will take on a value within two standard deviation of the mean and compare the probability with the lower bound provided by Tchebyshev theorem. 3

(c) Let X be the number of times that a fair coin is flipped 28 times lands on tails. Find the probability that X=14. Use the normal approximation and then compare it with the exact solution. The value of $\Phi(0.19) = 0.5753$, where $\Phi(z)$ is the cumulative distribution function of standard normal variate. 3