BACHELOR OF SCIENCE EXAMINATION, 2018

(3rd Year, 2nd Semester)

MATHEMATICS (Honours)

Paper - 6.3

(Algebra - IV)

Full Marks : 50 Time : Two Hours

Symbols / Notations have their usual meanings.

Use a separate Answer-script for each part.

Part - I

(Marks - 25)

Answer any *five* questions.

All questions carry equal marks.

1. Let G be a group. Define the group of automorphisms of G and inner automorphisms of G. Show that the set of all inner automorphisms of G is a normal subgroup of the group of auto automorphisms of G.

[*Turn over*]

2. Let G be a group. Define the center $Z(G)$ of G. Show that $G/Z(G) \simeq I(G)$ where $I(G)$ is defined by group of inner automorphisms of G.

3. If G is a finite abelian group of order n and m is a positive divisor of n, then show that G has a subgroup of order m.

4. Define simple group. Let p and q be two prime numbers. Prove that any group of order pq is not simple.

5. Suppose G be a group of order p^k, where $k \geq 1$ and p is prime. Then show that $|Z(G)| > 1$.

6. Define the torsion subgroup $T(G)$ of a finitely generated abelian group G.

   Let $G = Z_4 \oplus Z \oplus Z_5 \oplus Z_3$. Find $|T(G)|$ and show that $T(G)$ is cyclic.

7. (a) Let G be a group of order 231. Show that G has a cyclic subgroup of order 77.

   (b) If a group G of order 52 containg a normal subgroup of order 4, show that G is a commutative group.

$2\frac{1}{2} \times 2\frac{1}{2}$

[*Turn over*]

Part - II

(Marks - 25)

Answer any *five* questions.

1. Define ideal of a ring. Show that any integral domain R with only a finite number of ideals is a field. Hence conclude that any finite integral domain is a field. 1+3+1=5

2. Define quotient ring. Let I be an ideal of a ring R. Show that the quotient ring R/I is a commutative ring if and only if $ab - ba \in I$ for all a, $b \in R$. Is quotient ring of an integral domain always an integral domain ? Justify your answer. 2+2+1=5

3. State and prove First Isomorphism Theorem of ring. 2+3=5

4. (i) Show that any epimorphism from the ring $\mathbb{Z}$ of integers onto itself is an isomorphism.

   (ii) Let $f : F \to F'$ be a nontrivial homomorphism from a field F onto a ring F'. Show that F' is a field. 3+2=5

5. (i) Let $\mathbb{R}$ be a Boolean ring with identity. Show that every prime ideal of $\mathbb{R}$ is a maximal ideal of $\mathbb{R}$.

[*Turn over*]

(ii) Let $\mathbb{R}$ be a finite commutative ring with identity. Is every prime ideal of $\mathbb{R}$ a maximal ideal of $\mathbb{R}$? Justify your answer. 3+2=5

6. Define Euclidean Domain (ED). Show that every Euclidean Domain (ED) is a Principal Ideal Domain (PID). 2+3=5

7. State Eisenstein criterion for irreducibility of a polynomial. Show that the polynomial $f(x) = 1 + x + x^2 + ... + x^{30}$ is irreducible in $Q[x]$, by using Eisenstein irreducibility criterion. 2+3=5