BACHELOR OF SCIENCE EXAMINATION, 2018

(3rd Year, 2nd Semester)

MATHEMATICS (Honours)

Paper - 6.5 (A)

(Mechanics - IV)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

Notations and Symbols have their usual meanings.

1. Answer any *five* questions : 8×5=40

(a) A circular tube with centre O is filled with three fluids of densities ρ_1, ρ_2, ρ_3 (in descending order of magnitude) and placed in a vertical plane. If $2\alpha, 2\beta, 2\gamma$ be the angles subtended at the centre by the fluids and P be the point on the circumference midway between the ends of the lightest fluid, then the angle θ which OP makes with the vertical is given by

$$\frac{\rho_2 - \rho_3}{\rho_1 - \rho_3} = \frac{\sin\alpha}{\sin(\alpha+\theta)}, \frac{\sin(\beta-\theta)}{\sin\beta}.$$

[*Turn over*]

(b) ABC is a triangle immersed vertically in water with C in the surface and the sides AC, BC equally inclined to the surface. Prove that the vertical through C divides the triangle into two others, the thrusts on which are in the ratio $b^3 + 3ab^2 : a^3 + 3a^2b$.

(c) A quadrilateral is immersed in water with two angular points in the surface and the other two at depths a and b. If α and β are the depths below the surface of its centre of gravity and centre of pressure respectively, then show that $6\alpha\beta + ab = 3\alpha(a+b)$.

(d) A solid sphere of density ρ is placed at the bottom of a vessel which is horizontal and a liquid of density $\sigma(<\rho)$ is poured in so as just to cover up the sphere. The sphere is then cut along the vertical diametrical plane. Prove that the two parts will not separate if $\rho < 4\sigma$.

(e) A vertical cylinder, of density ρ, floats in two liquids, the density of the upper liquid being ρ_1 and that of the lower ρ_2. If the length h of the cylinder be n times the depth of the upper liquid, prove that the depth of immersion of the upper face of the cylinder is

$$\frac{h}{n} - h\left(\frac{\rho_2 - \rho}{\rho_2 - \rho_1}\right),$$

[*Turn over*]

provided $\rho_2 - \frac{(\rho_2 - \rho_1)}{n} < \rho < \rho_2$.

(f) A hollow cone very nearly filled with weater, rotates uniformly about its axis which is vertical, the vertex being uppermost. If the pressure on the base be equal to six times the weight of the enclosed water, prove that the angular velocity is $\sqrt{\frac{4g}{a}\cot\alpha}$, where a is the radius of the base and 2α is the vertical angle of the cone.

(g) A solid paraboloid of *sp.gr.*σ floats in a liquid of *sp.gv.*ρ with its axis vertical and vertex downwards. Show that the equilibrium will be stable if

$$3a > h\left(1 - \sqrt{\frac{\sigma}{\rho}}\right)$$

where $4a$ is the latus rectum of the generating parabola and h is the height of the solid.

2. Answer any *one* question : 10×1

(a) (i) Obtain the necessary condition for the equilibrium of a mass of liquid under a given system of forces whose components per unit of mass at the point

(x, y, z) parallel to the co-ordinate axes are (F_x, F_y, F_z). 5

(ii) A given volume V of a liquid is acted on by forces $-\frac{\mu x}{a^2}, -\frac{\mu y}{b^2}, -\frac{\mu z}{c^2}$ per unit mass at (x, y, z) parallel to the axes. Find the equation of the free surface. 5

(b) For a body floating freely in a homogeneous liquid at rest under gravity prove that $HM = {AK^2}/{V}$, with usual notation. 10

———————