Ex/Math/H/32/6.5/B/86/2018

BACHELOR OF SCIENCE EXAMINATION, 2018

(3rd Year, 2nd Semester)

MATHEMATICS (Honours)

Paper - 6.5 (B)

(Differential Geometry)

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

(Symbols / Notations have their usual meanings.)

Answer any *five* questions. 10×5=50

1. (a) Obtain the equation of the curve $\vec{r} = (a\cos t, a\sin t, bt)$, where a, b are constants, $(a > 0)$ and $-\infty < t < \infty$, referred to 's' (are length) as parameter and show that the length of one complete turn of the curve is $2\pi c$, where $c = \sqrt{a^2 + b^2}$. Also identify the curve.

   (b) Find the curvature and torsion of the above curve. 4+6=10

[*Turn over*]

2. (a) Find the equations of osculating plane, normal plane and rectifying plane to the space curve $x = t,\ y = t^2,\ z = t^3$ at $t = 1$.

   (b) Find the curvature k of the space curve $\vec{r} = r(t)$, is given by $\vec{r}(t) = (t, t^2, t^3)$. 6+4=10

3. State and prove fundamental theorem for space curve. 10

4. (a) Prove that for Bertrand mates the product of the torsions is constant.

   (b) Find the parametric curves of a surface $\vec{r} = (u\cos v,\ u\sin v,\ cv)$ and identify them, c being constant. 6+4=10

5. (a) Deduce the first fundamental form of a surface and find it for the surface $\vec{r} = (u + v,\ 1 - uv,\ u - v)$.

   (b) Also find the angle between the parametric curves and unit normal vector for the above mentioned surface. 5+5=10

6. (a) If a surface is represented by $z = f(x, y)$ then obtain the expression for first fundamental form and unit normal vector to the surface.

(b) Find the area enclosed by the parametric curves on a surface $\vec{r} = (u\cos v, u\sin v, 3u)$, for $u = 1$ to $u = 4$ and $v = 0$ to $v = \frac{\pi}{4}$. 4+6=10

7. (a) Obtain the necessary and sufficient condition for parametric curves on a surface to be orthogonal. Hence check whether the parametric curves on a sphere are orthogonal or not.

(b) Prove that if for a space curve the ratio of curvature and torsion is constant then it is a helix. 5+5=10