Ex/Math/H/32/6.6/B/87/2018

BACHELOR OF SCIENCE EXAMINATION, 2018

(3rd Year, 2nd Semester)

MATHEMATICS (Honours)

Paper - 6.6 (B)

[Discrete Mathematics-II]

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.

(Notations and Symbols have their usual meanings, if not mentioned otherwise)

1. Attempt any *two* of the following ? 6×2=12

 (a) Give a DFA accepting the language $\{a^m b^n \mid m \geq 1, n \geq 1 \text{ and } 3 \text{ divides } m+n\}$ over the alphabet $\{a, b\}$.

 (b) Construct a finite automaton equivalent to the regular expression $(1+01+001)^*(\in +0+00)$.

[*Turn over*]

(c) Consider the grammar $G=(V,T,P,S)$ where $V=\{S,A,B\}$, $T=\{a,b\}$, P has the productions :

$$S \rightarrow aB \mid bA,$$
$$A \rightarrow a \mid aS \mid bAA,$$
$$B \rightarrow b \mid bS \mid aBB.$$

For the string *aaabbabbba* find a (i) leftmost derivation, (ii) rightmost derivation, and (iii) parse tree. Is the grammar unambiguous ?

2. Attempt any *two* of the following : 6×2=12

(a) Using the pumping lemma, verify if the language $L = \{0^n \mid n \text{ is prime}\}$ is regular or not.

(b) Let h be a homomorphism defined by $h(a)=01,\ h(b)=0.$ Using usual extension of h

(i) find $h^{-1}(L_1)$, where $L_1=(10+1)^*$

(ii) find $h^{-1}(L_2)$, where $L_2=(a+b)^*$

(c) Let $L(M)$ be the language accepted by a finite automaton M with n states. Prove that $L(M)$ is infinite if and only if M acceptes some sentence of length l, where $n \leq l < 2n.$

[*Turn over*]

3. Attempt any *one* of the following : 8

(a) Let $G = (\{S, A, B\}, \{a, b\}, P, S)$ be the grammar when P consists of the following productions :

$S \rightarrow AB$

$A \rightarrow aAA \mid \in$

$B \rightarrow bBB \mid \in$

Construct a *CFG* with no $\in$-production accepting $L(G) - \{\in\}$.

(b) Given a grammar $G = (V, T, P, S)$ such that $V = \{S\}$, $T = \{p, q, n, b, e, c\}$, P consists of the productions $S \rightarrow nS \mid bScSe \mid p \mid q$ find an equivalent grammar in CNF.

4. Attempt any *one* of the following : 9

(a) Construct a PDA accepting $L(G)$ by empty stack where $G = (\{S\}, \{0, 1, c\}, P, S)$ with $P = \{S \rightarrow 0S0 \mid 1S1 \mid c\}$.

(b) Let $M_1 = \left(\{q_0, q_1\}, \{0,1\}, \{X, Z\}, \delta, q_0, Z, \emptyset\right)$ be a PDA where δ is given by

$$\begin{aligned}
\delta(q_0, 0, Z) &= \{(q_0, X Z)\}, \\
\delta(q_1, 1, X) &= \{(q_1, \in)\}, \\
\delta(q_0, 0, X) &= \{(q_0, X X)\}, \\
\delta(q_0, 1, X) &= \{(q_1, \in)\}, \\
\delta(q_1, \in, X) &= \{(q_1, \in)\}, \\
\delta(q_1, \in, Z) &= \{(q_1, \in)\}.
\end{aligned}$$

Construct a *PDA* accepting $N(M_1)$ by final states.

5. (a) What is Chomsky hierarchy. 2

(b) Attempt any *one* of the following : 7

(i) Given the regular expression (((01+10) ∗ 11) ∗ 00)∗ on {0, 1}, construct an equivalent regular grammar.

(ii) Construct a deterministic finite automaton equivalent to the grammar $G = \left(\{S, A, B\}, \{0,1\}, P, S\right)$ where $P = \{S \to A0,\ A \to AB0 \mid 1,\ B \to 1\}$.
